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THE STABILITY OF OUT-OF-PLANE EQUILIBRIUM POINTS IN THE ELLIPTIC RESTRICTED THREE-BODY PROBLEM AT J_4 OF THE BIGGER PRIMARY

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ABSTRACT

This paper examines the positions and stability of out-of-plane equilibrium points within the framework of the Elliptic Restricted Three-Body Problem (ER3BP) at J_4 of the bigger primary in the field of stellar binary systems: HD188753 and Alpha Centauri around their common centre of mass in elliptic orbits. It is found that, the positions and stability of the out-of-plane equilibrium points are significantly affected by the oblateness, semi-major axis and the eccentricity of their orbits. The positions $L_{6,7}$ of the infinitesimal body lie in the xz −plane almost directly above and below the center of the bigger oblate primary. The effects of the perturbed parameters on the positions and stability are shown numerically for the aforementioned binary systems. The status of the out-of-plane equilibrium points as evidenced in most cases remain the same and are unstable even when only the bigger primary is viewed in the out-of-plane.

Keywords: Out-Of-Plane, Stability, Elliptic Restricted Three-Body Problem), J_4, Bigger primary.

INTRODUCTION

The restricted three-body problem (R3BP) is a problem that has continue to be of great theoretical, practical, historical and educational importance. The study of this problem have had important implications in various scientific fields, some of which includes; celestial mechanics, chaos theory, galactic dynamics, molecular physics and many others. This problem is still a stimulating and active research field that is receiving considerable attention of scientists and astronomers due to its applications in dynamics of the stellar and solar systems, artificial satellites and lunar theory.

In the general restricted three-body problem, celestial bodies are assumed to be spherical, but in nature, several celestial bodies have observed the significant effects of oblateness on their bodies (Singh and Umar 2012, Tkhai 2012, Singh and Leke 2012, 2014, Douskos and Markel-los 2006, SubbaRao and Sharma 1975).

The elliptic restricted three-body problem describes the three-dimensional motion of an infinitesimal mass under the gravitational attraction of two primaries, which revolve on elliptic orbits in a plane around their common centre of mass. This is exemplified in the motion of an asteroid under the gravitational attraction of Sun and Jupiter.

Over the years, the restricted three-body problem with oblateness of the primaries have recorded rapid growth, specifically in the two and three-dimensional cases with respect to its five co-planar equilibrium points $L_i(i = 1, \ldots, 5)$: The points L_1, L_2, L_3 lying on the line joining the primaries are called collinear equilibrium points, while the points L_4, L_5 forming the triangle with the line joining the primaries are called triangular points.

Numerous researches have affirmed the instability of collinear points in most cases (Singh and Umar 2012, 2013, Ammar 2012, Tsirogiannis et al. 2006, Sharma 1987). Singh and Leke (2012) studied the equilibrium points and their stability in the R3BP with oblateness and variable masses. Their result shows that, the collinear points are stable due to k (kappa). However, it remains unstable in the out-of-plane equilibrium points despites the introduction of a small perturbation in the centrifugal force, radiation pressure, oblateness of the first primary and k. Singh and Leke (2014) examined the analytic and numerical treatment of motion of dust grain particle around triangular equilibrium points with post-AGB binary star and disc. They concluded that, the triangular points around IRAS 11472-0800-G29-38 system are particularly unstable.

The positions and stability of triangular and collinear equilibrium points were seem to be affected by the oblateness of the primaries, semi-major axis and eccentricity of their orbits, which leads to a decrease in the size of the region of stability with an increase in the parameters involved (Singh and Umar 2012, 2013; Singh and Tyokyaa 2016, 2017).

The equation of motion of the three-dimensional restricted three-body problem with oblateness of the primaries allows the existence of out-of-plane equilibrium points. These points lie in the xz −plane symmetrically with respect to the $x-axis$ along the curve almost directly above and below the centre of each oblate primary. These points are denoted by $L_{6,7}$ (Abouelmagd 2012; Das et al. 2009; Singh and Umar 2012, 2013; Singh and Amuda 2015).

Das et al. (2009) observed that, in the photo-gravitational circular restricted three-body problem, the out-of-plane equilibrium points are of a passive micro size particle when their stability in the field of radiating binary systems are considered. Singh and Amuda (2015) studied the out-of-plane equilibrium points with Poynting-Robertson (P-R) drag, they found that due to the ratio of radiation to the gravitational force of the smaller primary and the expression of y_o coordinate with oblateness of the bigger primary. The out-of-plane equilibrium points exist but its stability analysis remain the same and are unstable.

Singh and Umar (2012) examined the motion of a particle under the influence of an oblate dark degenerate primary, a luminous secondary and the stability of triangular points when both oblate primaries emit light energy simultaneously in the elliptic restricted three-body problem respectively. They found that, in the stellar systems, a planet moving in the field of a binary star system effectively constitutes a three-body system.

Our study focuses on the stability of out-of-plane equilibrium points within the framework of the Elliptic Restricted Three-Body Problem (ER3BP) at J_4 of the bigger primary in the field

of stellar binary systems: HD188753 and Alpha Centauri around their common center of mass in elliptic orbits.
The paper is organized as follows: Sections 2 presents the equations of motion; section 3 locates the positions of out-of-plane equilibrium points; sections 4 examines their stability; section 5 explores numerical application; the discussions and conclusions are provided in section 6.

EQUATION OF MOTION

The equations of motion of an infinitesimal mass in the elliptic restricted three-body problem under the influence of the oblate bigger primary at J_4 are adopted from Tyokyaa and Bichi (2016) and are presented in dimensionless-pulsating coordinate system (ξ, η, ζ) as follows;

$$\xi'' - 2\eta' = \frac{\delta\Omega}{\delta\xi}, \qquad \eta'' + 2\xi' = \frac{\delta\Omega}{\delta\eta}, \qquad \zeta'' = \frac{\delta\Omega}{\delta\zeta} \tag{1}$$

given the force function as

$$\Omega = (1-e^2)^{-\frac{1}{2}}\left[\frac{1}{2}(\xi^2+\eta^2) + \frac{1}{n^2}\left\{\frac{(1-\mu)}{r_1} + \frac{(1-\mu)A_1}{2{r_1}^3} - \frac{3(1-\mu)A_2}{8{r_1}^5} - \frac{3(1-\mu)A_1Z^2}{2{r_1}^5} + \frac{9(1-\mu)A_2Z^2}{8{r_1}^7} + \frac{\mu}{r_2}\right\}\right] \tag{2}$$

The mean motion, n, is given as

$$n^2 = \frac{(1+e^2)^{\frac{1}{2}}}{a(1-e^2)}\left[1 + \frac{3}{2}A_1 - \frac{15}{8}A_2\right] \tag{3}$$

$${r_i}^2 = (\xi - \xi_i)^2 + \eta^2 + \zeta^2, (i = 1,2) \quad \xi_1 = -\mu, \ \ \xi_2 = 1-\mu\,, \ \ \mu = \frac{m_2}{m_1+m_2} \tag{4}$$

where, m_1, m_2 are the masses of the bigger and smaller primaries positioned at the points $(\xi_i, 0, 0), i = 1{,}2$; $A_i = J_{2i}{R_1}^2$ characterize zonal harmonic oblateness of the bigger primary whose mean radiiis R_1 . $\mu = \frac{m_2}{m_1+m_2}$ is the mass ratio, while a and e are the semi-major axis and eccentricity of the orbits, respectively.

Positions of out-of-plane equilibrium points

To locate the positions of the out-of-plane equilibrium points denoted by $L_{6,7}$, we solve for the solutions of $\Omega_\xi = \Omega_\eta = \Omega_\zeta = 0$ but;

$$\Omega_\xi = (1-e^2)^{-\frac{1}{2}}\left[\xi - \frac{1}{n^2}\left(\frac{(1-\mu)(\xi+\mu)}{{r_1}^3} + \frac{3(1-\mu)(\xi+\mu)A_1}{2{r_1}^5} - \frac{15(1-\mu)(\xi+\mu)A_2}{8{r_1}^7} - \frac{15(1-\mu)(\xi+\mu)A_1Z^2}{2{r_1}^7} + \frac{63(1-\mu)(\xi+\mu)A_2Z^2}{8{r_1}^9} + \frac{\mu(\xi+\mu-1)}{{r_2}^3}\right)\right] \tag{5}$$

$$\Omega_\eta = (1-e^2)^{-\frac{1}{2}}\left[\eta\left(1 - \frac{1}{n^2}\left(\frac{(1-\mu)}{{r_1}^3} + \frac{3(1-\mu)A_1}{2{r_1}^5} - \frac{15(1-\mu)A_2}{8{r_1}^7} - \frac{15(1-\mu)A_1Z^2}{2{r_1}^7} + \frac{63(1-\mu)A_2Z^2}{8{r_1}^9} + \frac{\mu}{{r_2}^3}\right)\right)\right] \tag{6}$$

$$\Omega_\zeta = \frac{(1-e^2)^{-\frac{1}{2}}}{n^2}\left[-\zeta\left(\frac{(1-\mu)}{{r_1}^3} + \frac{3(1-\mu)A_1}{2{r_1}^5} - \frac{15(1-\mu)A_2}{8{r_1}^7} - \frac{15(1-\mu)A_1Z^2}{2{r_1}^7} + \frac{63(1-\mu)A_2Z^2}{8{r_1}^9} + \frac{\mu}{{r_2}^3}\right)\right] \tag{7}$$

then;

$$\xi - \frac{1}{n^2}\left(\frac{(1-\mu)(\xi+\mu)}{{r_1}^3} + \frac{3(1-\mu)(\xi+\mu)A_1}{2{r_1}^5} - \frac{15(1-\mu)(\xi+\mu)A_2}{8{r_1}^7} - \frac{15(1-\mu)(\xi+\mu)A_1Z^2}{2{r_1}^7} + \frac{63(1-\mu)(\xi+\mu)A_2Z^2}{8{r_1}^9} + \frac{\mu(\xi+\mu-1)}{{r_2}^3}\right) = 0 \tag{8}$$

$$\eta\left(1 - \frac{1}{n^2}\left(\frac{(1-\mu)}{{r_1}^3} + \frac{3(1-\mu)A_1}{2{r_1}^5} - \frac{15(1-\mu)A_2}{8{r_1}^7} - \frac{15(1-\mu)A_1Z^2}{2{r_1}^7} + \frac{63(1-\mu)A_2Z^2}{8{r_1}^9} + \frac{\mu}{{r_2}^3}\right)\right) = 0 \tag{9}$$

$$-\zeta\left(\frac{(1-\mu)}{{r_1}^3} + \frac{3(1-\mu)A_1}{2{r_1}^5} - \frac{15(1-\mu)A_2}{8{r_1}^7} - \frac{15(1-\mu)A_1Z^2}{2{r_1}^7} + \frac{63(1-\mu)A_2Z^2}{8{r_1}^9} + \frac{\mu}{{r_2}^3}\right) = 0 \tag{10}$$

for the solutions of equations (8) and (10) with $\eta = 0$ and $\zeta \neq 0$ equation (10) becomes

$$\frac{-(1-\mu)}{n^2{r_1}^3} - \frac{3(1-\mu)A_1}{2n^2{r_1}^5} + \frac{15(1-\mu)A_2}{8n^2{r_1}^7} + \frac{15(1-\mu)A_1Z^2}{2n^2{r_1}^7} - \frac{63(1-\mu)A_2Z^2}{8n^2{r_1}^9} - \frac{\mu}{n^2{r_2}^3} = 0 \tag{11}$$

Multiplying equation (11) by $\xi - \xi_1$ and $\xi - \xi_2$ where $\xi_1 = -\mu$ and $\xi_2 = 1-\mu$ we have respectively;

$$\frac{-(1-\mu)(\xi+\mu)}{n^2{r_1}^3} - \frac{3(1-\mu)(\xi+\mu)A_1}{2n^2{r_1}^5} + \frac{15(1-\mu)(\xi+\mu)A_2}{8n^2{r_1}^7} + \frac{15(1-\mu)(\xi+\mu)A_1Z^2}{2n^2{r_1}^7} - \frac{63(1-\mu)(\xi+\mu)A_2Z^2}{8n^2{r_1}^9} - \frac{\mu(\xi+\mu)}{n^2{r_2}^3} = 0 \tag{12}$$

$$\frac{-(1-\mu)(\xi+\mu-1)}{n^2{r_1}^3} - \frac{3(1-\mu)(\xi+\mu-1)A_1}{2n^2{r_1}^5} + \frac{15(1-\mu)(\xi+\mu-1)A_2}{8n^2{r_1}^7} + \frac{15(1-\mu)(\xi+\mu-1)A_1Z^2}{2n^2{r_1}^7} - \frac{63(1-\mu)(\xi+\mu-1)A_2Z^2}{8n^2{r_1}^9} - \frac{\mu(\xi+\mu-1)}{n^2{r_2}^3} = 0 \tag{13}$$

Subtracting equation (12) from (8) and substituting the value of $n^2 = \frac{1}{a}\left[1 + \frac{3}{2}A_1 - \frac{15}{8}A_2 + \frac{3e^2}{2}\right]$ yields;

$$\xi = -\mu\left[1 + \frac{3A_1}{2} - \frac{15A_2}{8}\right] \tag{14}$$

From (4) we have

$$\zeta^2 = r_1^2 - (\xi+\mu)^2, (i = 1,2) \quad \xi_1 = -\mu, \ \ \xi_2 = 1-\mu\,, \ \ \mu = \frac{m_2}{m_1+m_2} \tag{15}$$

but from Singh and Tyokyaa (2016) we have;

$$r_1^2 = a^{2/3}\left(1 - e^2 - A_1 + \frac{5A_2}{4} + A_1a^{-2/3} - \frac{5A_2a^{-4/3}}{4}\right), r_2^2 = a^{2/3}\left(1 - e^2 - A_1 + \frac{5A_2}{4}\right) \tag{16}$$

Considering equations (15) and (16) yields

$$\zeta^2 = \left[a^{2/3}\left(1 - e^2 - A_1 + \frac{5A_2}{4} + A_1a^{-2/3} - \frac{5A_2a^{-4/3}}{4}\right) - \mu^2\left(1 + 3A_1 - \frac{15A_2}{4}\right)\right]$$

$$\zeta = \left[a^{2/3}\left(1 - e^2 - A_1 + \frac{5A_2}{4} + A_1a^{-2/3} - \frac{5A_2a^{-4/3}}{4}\right) - \mu^2\left(1 + 3A_1 - \frac{15A_2}{4}\right)\right]^{1/2} \tag{17}$$

Equations (14) and (17) are the positions of the out-of-plane equilibrium points denoted by $L_{6,7}$ for the study under review.

Stability of out-of-plane equilibrium points

The stability of the motion of a body in the vicinity of any of the out-of-plane points is obtained by establishing the characteristics equation of the study under review.

Let the location of any of the equilibrium point be denoted by (ξ_o, η_o, ζ_o) and suppose the small displacement of the location are (σ, β, α), then

$\xi = \xi_o + \sigma,\ \eta = \eta_o$ and $\zeta = \zeta_o + \alpha$

Taking derivatives, we have

$$\xi' = \sigma', \xi'' = \sigma'', \eta' = \beta', \eta'' = \beta'' \text{ and } \zeta' = \alpha', \zeta'' = \alpha'' \tag{18}$$

Given the equations of motion of the infinitesimal mass by Tyokyaa and Bichi (2016) as;

$$\xi'' - 2\eta' = \frac{\partial\Omega}{\partial\xi}, \eta'' - 2\xi' = \frac{\partial\Omega}{\partial\eta} \text{ and } \zeta'' = \frac{\partial\Omega}{\partial\zeta} \tag{19}$$

We obtain the characteristics equation of the system as;

$$\lambda^6 + \left(4 - \Omega^0_{\xi\xi} - \Omega^0_{\eta\eta} - \Omega^0_{\zeta\zeta}\right)\lambda^4 + \left(\Omega^0_{\xi\xi}\Omega^0_{\eta\eta} + \Omega^0_{\eta\eta}\Omega^0_{\zeta\zeta} + \Omega^0_{\xi\xi}\Omega^0_{\zeta\zeta} - 4\Omega^0_{\zeta\zeta} - \left(\Omega^0_{\xi\zeta}\right)^2\right)\lambda^2 - \left(\Omega^0_{\xi\xi}\Omega^0_{\eta\eta}\Omega^0_{\zeta\zeta} - \left(\Omega^0_{\xi\zeta}\right)^2\Omega^0_{\eta\eta}\right) = 0 \tag{20}$$

The superscripts o shows that the partial derivatives are evaluated at the out-of-plane points under consideration. At the points under consideration ignoring products and higher order terms of very small parameters we have;

$$\Omega^0_{\xi\xi} = (1-e^2)^{-1/2}\left[1 - \frac{3(1-\mu)}{4a^{2/3}} - \frac{3\mu}{4a^{2/3}} - \frac{3(1-\mu)A_1}{8a^{2/3}} + \frac{21\mu A_1}{8a^{2/3}} - \frac{3(1-\mu)e^2}{4a^{2/3}} - \frac{3\mu e^2}{4a^{2/3}} + \frac{105(1-\mu)A_2}{32a^2} + \frac{105(1-\mu)A_1Z^2}{8a^2} - \frac{45(1-\mu)A_2}{32a^{2/3}} - \frac{45\mu A_2}{32a^{2/3}}\right] \tag{21}$$

$$\Omega^0_{\eta\eta} = (1-e^2)^{-1/2}\left[\frac{9(1-\mu)}{4a^{2/3}} + \frac{9\mu}{4a^{2/3}} - \frac{39(1-\mu)A_1}{8a^{2/3}} - \frac{39\mu A_1}{8a^{2/3}} - \frac{3(1-\mu)e^2}{4a^{2/3}} - \frac{3\mu e^2}{4a^{2/3}} + \frac{195(1-\mu)A_2}{32a^{2/3}} + \frac{195\mu A_2}{32a^{2/3}} - \frac{315(1-\mu)A_2}{32a^2} - \frac{315(1-\mu)A_1Z^2}{8a^2}\right] \tag{22}$$

$$\Omega^0_{\zeta\zeta} = (1-e^2)^{-1/2}\left[\frac{3\mu}{2a^{2/3}} - \frac{3(1-\mu)}{2a^{2/3}} + \frac{21(1-\mu)A_1}{4a^{2/3}} - \frac{9\mu A_1}{4a^{2/3}} - \frac{45(1-\mu)A_2}{16a^{2/3}} + \frac{45\mu A_2}{16a^{2/3}} + \frac{105(1-\mu)A_2}{16a^2} - \frac{3(1-\mu)e^2}{2a^{2/3}} + \frac{3\mu e^2}{2a^{2/3}} + \frac{105(1-\mu)A_1Z^2}{4a^2}\right] \tag{23}$$

$$\Omega^0_{\xi\zeta} = (1-e^2)^{-1/2}\left[-\frac{3(1-\mu)}{8a^{2/3}} - \frac{3\mu}{8a^{2/3}} + \frac{51(1-\mu)A_1}{16a^{2/3}} + \frac{15\mu A_1}{16a^{2/3}} - \frac{45(1-\mu)A_2}{64a^{2/3}} - \frac{45\mu A_2}{64a^{2/3}} + \frac{105(1-\mu)A_2}{64a^2} - \frac{3(1-\mu)e^2}{8a^{2/3}} - \frac{3\mu e^2}{8a^{2/3}} + \frac{105(1-\mu)A_1Z^2}{16a^2}\right] \tag{24}$$

Substituting equations (21)-(24) and neglecting higher order terms of very small parameters we have;

$$\lambda^6 + P\lambda^4 + Q\lambda^2 - R = 0 \tag{25}$$

Where;

$$P = \frac{11}{2} + 2\mu + 3\mu\alpha + \left\{\frac{3}{2} - \frac{45\mu}{4}\right\}A_1 + \left\{-\frac{3}{4} + 4\mu\right\}e^2 + \left\{\frac{135}{16} + \frac{15\mu}{8}\right\}A_2 + \left\{-\frac{105}{4} + \frac{105\mu}{4}\right\}A_1Z^2 \tag{26}$$

$$Q = \frac{27}{4} - 9\mu + \left\{\frac{9}{2} - 6\mu\right\}\alpha + \left\{-\frac{563}{32} + \frac{623\mu}{32}\right\}A_1 + \left\{\frac{15}{2} - 12\mu\right\}e^2 + \left\{-\frac{585}{32} + \frac{510\mu}{32}\right\}A_2 + \left\{-\frac{945}{8} + \frac{945\mu}{8}\right\}A_1Z^2 \tag{27}$$

$$Q = -\frac{81}{256} + \left\{-\frac{81}{128}\right\}\alpha + \left\{\frac{3{,}105}{512} - \frac{243\mu}{64}\right\}A_1 + \left\{-\frac{513}{512} + \frac{540\mu^3}{256}\right\}e^2 + \left\{-\frac{4{,}185}{2{,}048}\right\}A_2 \tag{28}$$

Now, equation (25) becomes;

$$\lambda^6 + \left[\frac{11}{2} + 2\mu + 3\mu\alpha + \left\{\frac{3}{2} - \frac{45\mu}{4}\right\}A_1 + \left\{-\frac{3}{4} + 4\mu\right\}e^2 + \left\{\frac{135}{16} + \frac{15\mu}{8}\right\}A_2 + \left\{-\frac{105}{4} + \frac{105\mu}{4}\right\}A_1Z^2\right]\lambda^4 + \left[\frac{27}{4} - 9\mu + \left\{\frac{9}{2} - 6\mu\right\}\alpha + \left\{-\frac{563}{32} + \frac{623\mu}{32}\right\}A_1 + \left\{\frac{15}{2} - 12\mu\right\}e^2 + \left\{-\frac{585}{32} + \frac{510\mu}{32}\right\}A_2 + \left\{-\frac{945}{8} + \frac{945\mu}{8}\right\}A_1Z^2\right]\lambda^2 - \left[-\frac{81}{256} + \left\{-\frac{81}{128}\right\}\alpha + \left\{\frac{3{,}105}{512} - \frac{243\mu}{64}\right\}A_1 + \left\{-\frac{513}{512} + \frac{540\mu^3}{256}\right\}e^2 + \left\{-\frac{4{,}185}{2{,}048}\right\}A_2\right] = 0 \tag{29}$$

NUMERICAL APPLICATIONS

Considering (14), (17) and (29), the positions and stability of the out-of-plane equilibrium points are computed numerically using the software package MATHEMATICA for the systems HD188753 and Alpha Centauri. The positions and stability of the out-of-plane equilibrium points for varying oblateness are given in Tables 2 and 3 to show the effects of the oblateness up to zonal harmonic J_4 of bigger primary, mass ratio, eccentricity of the orbits and the semi-major axis. We considered $a = 1 - \alpha$ and $\alpha \ll 1$ in the computation. These effects are also demonstrated graphically in Figures 1-4. The effects of the parameters on the positions and the stability region of the study under review are shown in Tables 2 and 3 for the systems: HD188753 and Alpha Centauri. As it is evidenced from Tables 2 and 3, for each set of values at least one root is a complex root with positive real part, hence in the Lyapunov sense, the stationary points are unstable. This agrees with the work of Douskos and Markellos (2006), Das et al. (2009), Singh and Umar (2012, 2013), Singh and Amuda (2015), Singh and Tyokyaa (2016, 2017), Tyokyaa and Bichi (2016), Tyokyaa et al (2017).

Table 1: Numerical data for the binary systems

Binary system	Masses m_1	Masses m_2	Mass ratio (μ)	Semi-major axis (a)	Eccentricity (e)
HD188753	1.13	1.01	0.4796	0.9837	0.5
Alpha Centauri	1.1000	0.907	0.4519	13.1236	0.51866

Table 2: Positions and Stability of out-of-plane points for HD188753 for $\mu = 0.4796, a = 0.9837, e = 0.5$ and $Z = 0.01$

Oblateness		Positions of out-of-plane points		Stability of out-of-plane points		
A_1	A_2	ξ	$\pm\zeta$	$\pm\lambda_{1,2}$	$\pm\lambda_{3,4}$	$\pm\lambda_{5,6}$
0.00	0.00	−0.4796	0.71541	$-0.179065 \pm 0.504331i$	$0 \pm 2.51599i$	$0.179065 \pm 0.504331i$
0.01	−0.005	−0.49129	0.707703	$-0.157739 \pm 0.498537i$	$0 \pm 2.49836i$	$0.157739 \pm 0.498537i$
0.02	−0.01	−0.502981	0.699912	$-0.130937 \pm 0.492174i$	$0 \pm 2.48058i$	$0.130937 \pm 0.492174i$
0.03	−0.015	−0.514671	0.692033	$-0.0935419 \pm 0.485117i$	$0 \pm 2.46264i$	$0.0935419 \pm 0.485117i$
0.04	−0.02	−0.526361	0.684063	$0 \pm 0.456212i$	$0 \pm 0.498178i$	$0 \pm 2.44455i$
0.05	−0.025	−0.538051	0.675999	$0 \pm 0.365073i$	$0 \pm 0.571237i$	$0 \pm 2.42629i$

Table 3: Positions and Stability of out-of-plane points for Alpha Centauri for $\mu = 0.4519, a = 13.1236, e = 0.51866$ and $Z = 0.01$

Oblateness		Positions of out-of-plane points		Stability of out-of-plane points		
A_1	A_2	ξ	$\pm\zeta$	$\pm\lambda_{1,2}$	$\pm\lambda_{3,4}$	$\pm\lambda_{5,6}$
0.00	0.00	−0.4519	1.96541	± 3.37919	$0 \pm 0.77257i$	$0 \pm 1.03653i$
0.01	−0.005	−0.462915	1.94258	± 3.39017	$0 \pm 0.77996i$	$0 \pm 1.02713i$
0.02	−0.01	−0.47393	1.91947	± 3.40113	$0 \pm 0.787859i$	$0 \pm 1.01723i$
0.03	−0.015	−0.484945	1.89608	± 3.41207	$0 \pm 0.796397i$	$0 \pm 1.00672i$
0.04	−0.02	−0.49596	1.8724	± 3.42299	$0 \pm 0.805767i$	$0 \pm 0.995393i$
0.05	−0.025	−0.506975	1.84841	± 3.43388	$0 \pm 0.816275i$	$0 \pm 0.982945i$

Graphs showing the effect of oblateness at J_4, the eccentricity and semi-major axis on the out-of-plane points for **HD188753** system

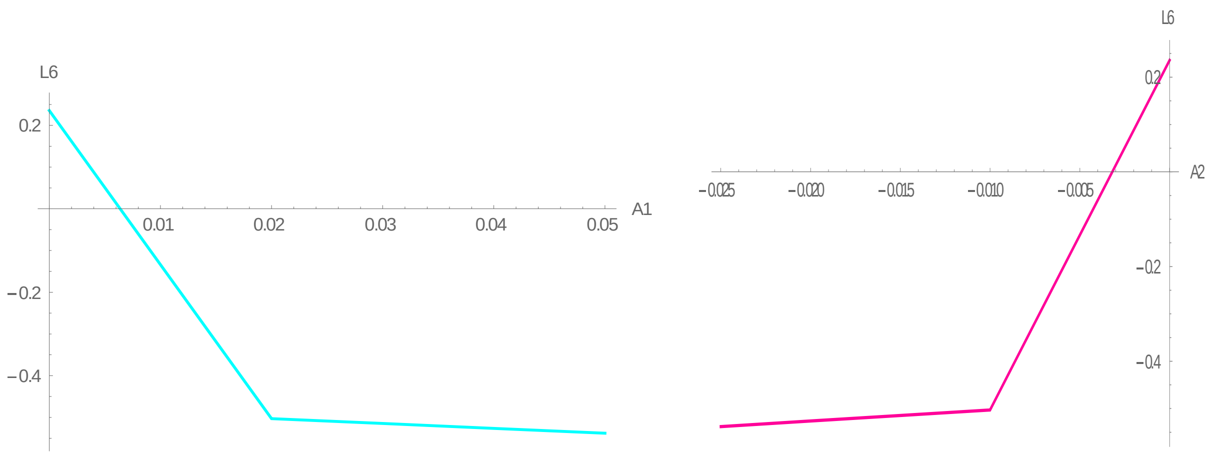

Fig. 1: Effects of oblateness at J_4 on L_6 for HD188753 system

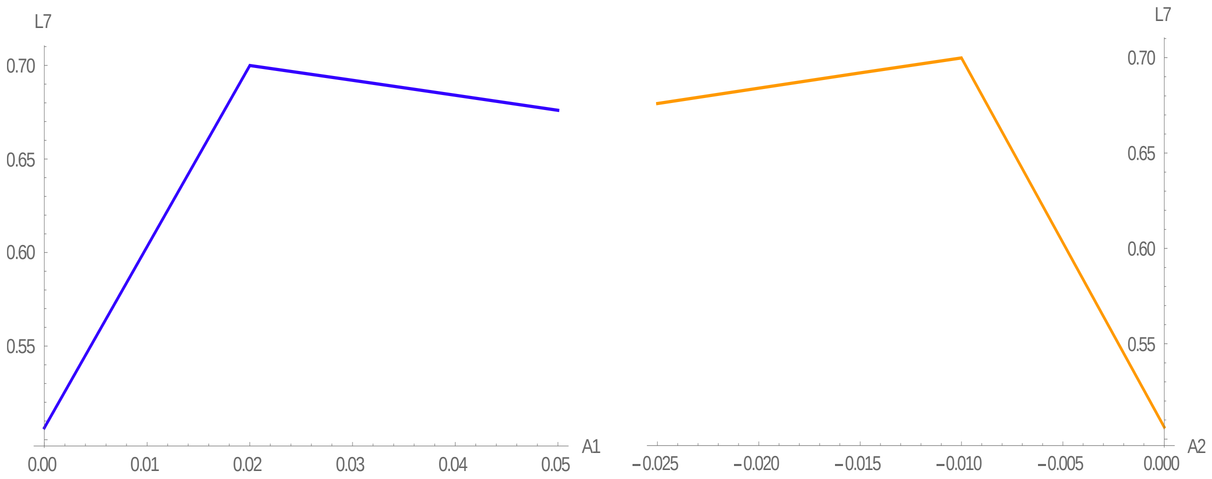

Fig 2: Effects of oblateness at J_4 on L_7 for HD188753 system

Graphs showing the effect of oblateness at J_4, the eccentricity and semi-major axis on the out-of-plane points for **Alpha Centauri** system

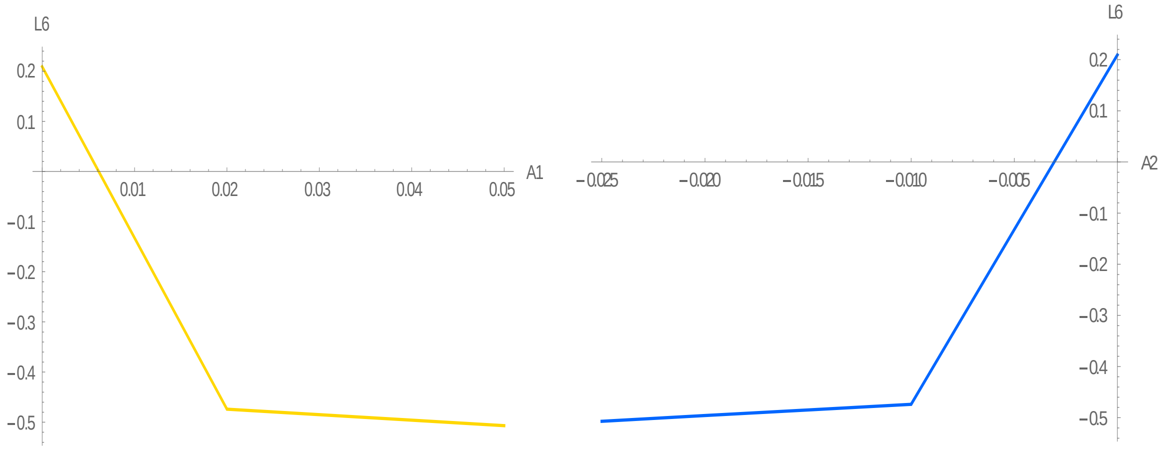

Fig 3: Effects of oblateness at J_4 on L_6 for Alpha Centauri system

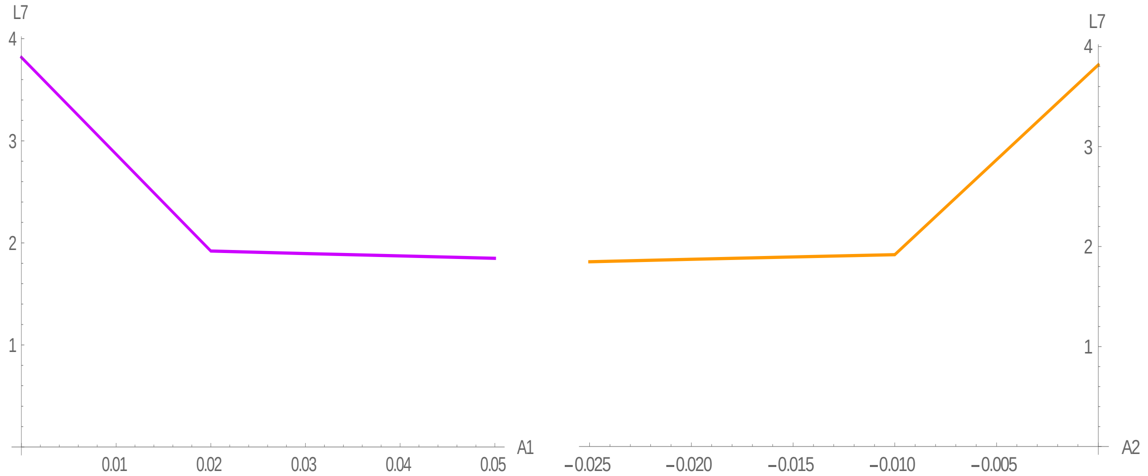

Fig. 4: Effects of oblateness at J_4 on L_7 for Alpha Centauri system

DISCUSSION AND CONCLUSION

The motion of the infinitesimal mass in the out-of-plane equilibrium points within the framework of the Elliptic Restricted Three-Body Problem (ER3BP) at J_4 of the bigger primary in the field of stellar binary systems: HD188753 and Alpha Centauri around their common center of mass in elliptic orbits is described in Equations 1-4. Equations 14 and 17 locate the positions of the out-of-plane equilibrium points of the bigger primary denoted by $L_{6,7}$. Our results coincides with Douskos and Markellos (2006) in the absence of eccentricity of the orbits, Zonal harmonics up to J_4 oblateness. As evidenced in the Tables 2 and 3 and Figures 1-4, the positions of the out-of-plane equilibrium points are affected by the eccentricity of the orbits, semi-major axis and oblateness of the bigger primary up to zonal harmonics J_4. This agrees with the result of Das et al. (2009), Singh and

Umar (2013), Singh and Amuda (2015). However, the perturbed oblateness does not change the nature of its stability. As shown in Tables 2 and 3, the status of the out-of-plane equilibrium points as evidenced in most cases remain the same and are unstable even when only the bigger primary is viewed in the out-of-plane.

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