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COMBINED EFFECTS OF VARIABLE VISCOSITY AND THERMAL RADIATION ON UNSTEADY NATURAL CONVECTION FLOW THROUGH A VERTICAL POROUS CHANNEL

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ABSTRACT
The present paper investigates on combined effects of variable viscosity and thermal radiation on unsteady natural convection flow. It is an extension of the work of Makinde *et al.* (2007) in which the fluid viscosity was considered to have a constant status. However, it is well known that the physical properties of fluids may change significantly with temperature. For lubricating fluids, heat generated by the internal friction and the corresponding rise in temperature affects the viscosity of the fluid and so the viscosity can no longer be assumed constant. Here, the fluid viscosity is considered to vary with temperature difference. The partial differential equations (PDEs) governing the fluid flow were transformed to dimensionless ordinary differential equations (ODEs) using similarity transformation. The obtained ODEs were solved using Adomian decomposition method (ADM). The effects of controlling physical parameters on the fluid temperature and its velocity are presented graphically and discussed. During the investigation; our result shows that, the velocity of the fluid increases with increase in viscosity variation parameter (λ) and thermal radiation while the fluid temperature increases with increase in thermal radiation. Notably, it is worthy to mention that; the result obtained herein coincides with that of Makinde *et al. (2007)* when λ and R tend to zero.

Keywords: Couette flow, Thermal radiation, Variable viscosity, Porous channel, ADM.

INTRODUCTION

One of the basic flows in fluid dynamics is Couette flow where the fluid motion is induced by the movement of the bounding surface. Fluid flow is either natural, forced or mixed convection. The natural convection flow is a flow induced by density difference occurring within the fluid particles due to temperature gradients. In this mechanism, the fluid surrounding the heat source receives heat, becomes less dense and raise, the surrounding cooler fluid then moves to replace it. The study of fluid flow with variable viscosity has become of principal interest in many scientific and engineering applications, such as crude oil extraction, petroleum industries, automobiles industries and so on. It is well known that the most sensitive fluid property to temperature rise is the viscosity. This was affirmed in the works of; John and Narayanan (1997), Hashemabadi *et al.* (2004), Becker and McKinley (2000). For many liquids, among them water, petroleum oils, glycerin, glycols, silicone fluids, and some molten salts, the percent variation of absolute viscosity with temperature is much more than that of the other properties. For instance, when the temperature increases from $10°C\ (\tilde{\mu} = 0.0133 g/cms)$ to $50°C(\tilde{\mu} = 0.00548 g/cms)$, the viscosity of water decreases by 240% (Carey and Mollendorf (1978)). The studies of Macosco (1994), Schlicting and Mahmud (2002) lamented that flow of viscous fluids with temperature dependent properties are of great importance in industries such as food processing, coating and polymer processing industries. In industrial systems, fluid can be subjected to extreme conditions such as high temperature, pressure and shear rate. External heating and high shear rate can lead to a high temperature being generated in the fluid. This may have a significant effect on the fluid properties. Fluids used in industries, such as polymer fluid has viscosity that varies rapidly with temperature change and may give rise to strong feedback effects. This consequently leads in the significant changes in the flow structure of the fluid (Sahin (1999)). When a system is under working conditions, some of its energy is wasted in the form of rays to the surrounding environment due to thermal radiation and this result in poor performance of the system.

Radiation effects on free convection flow are important in the context of space technology and processes involving high temperature. This is due to the safety of lives and properties especially in working medium that requires liberation of heat to the environment. In order to minimize thermal radiation emission, Makinde (2008) and Ibanez *et al.*, (2003) lamented that; when entropy generation takes place, the quality of energy of the system decreases. In a related literature, Ajibade *et al.* (2011) concluded that entropy generation increase with suction on one plate and decrease on the other plate with injection while Elbasbeshy and Bazid (2000) investigated the effect of temperature dependent viscosity on heat transfer over a moving surface where they assumed the fluid viscosity to vary as an inverse linear function of temperature. Costa and Macedonio (2003) applied temperature dependent viscosity model to study magma flows whereas Makinde and Ogulu (2011) analyzed the effect of thermal radiation on the heat and mass transfer flow of a variable viscosity fluid past a vertical porous plate permeated by a transverse magnetic field where they concluded that an increase in the positive value of viscosity variation parameter resulted into a decrease in the fluid viscosity. In the study of the later researchers, they used a linearized form of temperature in the radiative heat flux of their energy equation. This was however condemned by Magyari and Pantokratoras (2011) arguing that, a linearized form of temperature does not depict the real heat emission or conduction in the energy characteristics of most boundary layer flows. They therefore proposed alternative approach in which the temperature is evaluated using normal differentiation. In all the studies, some solution methods, such as variational iteration method (VIM), Runge-Kutta method, homotopy perturbation method (HPM), finite difference method were deployed for the solution of their flow problems. Other researchers have used Adomian decomposition method (ADM) proposed by Adomian (1994). With the advent of ADM, numerous scholars like Jiya and Oyubu (2012), Adesanya *et al.*, (2015), Adesanya and Makinde (2017), Adesanya and Gbadeyan (2010) and Venkatarangan and Rajalaksm (1995) have used the technique.

In the present study, the idea proposed by Magyari and Pantokratoras (2011) is adopted to analyse the flow equations and the fluid viscosity is assumed to vary linearly with temperature difference following Carey and Mollendorf (1978).

MATHEMATICAL PROBLEM

The physical problem under consideration consists of a vertical channel formed by two infinite vertical parallel porous plates; stationed h distance apart. The channel is filled with a viscous incompressible fluid in the presence of an incidence radiation flux of intensity q_r, which is absorbed by the plates and transferred to the fluid as shown in Figure 1. Similarly, the fluid physical properties are assumed to be constant except for its viscosity which is temperature dependent; also the fluid is considered optically thick where the radiative heat flux derived using Rosseland approximation can be utilized. The stream wise coordinate is denoted by x' taken along the channel in the vertically upwards direction and that normal to it is denoted by y'. The flow is assumed to be fully developed means that the axial (x' - direction) velocity depends only on the transverse co-ordinate y'. In addition, the effects of radiative heat flux in the x' - direction is considered negligible in comparison with that in the y' - direction. At time $t \le 0$, both the fluid and the plates are assumed to be at rest at constant temperature T_0. At time $t > 0$, the temperature of the plate kept at $y' = 0$ rise to T_w while the other plate at h distance from it, is fixed and maintained at temperature T_0. Since the plates are of infinite length, the velocity and temperature are functions of y' and t only.

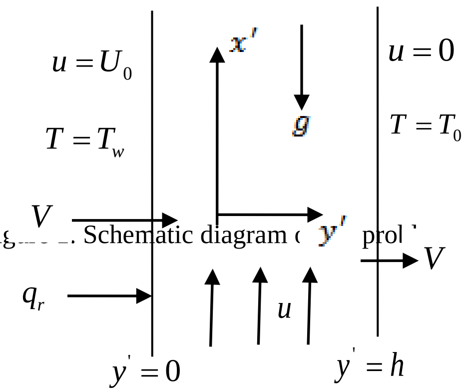

Fig... Schematic diagram ... prol...

Under these assumptions, the appropriate governing equations for the present problem in dimensional form are:

$$\frac{\partial u}{\partial t} + V_0 \frac{\partial u}{\partial y'} = \frac{1}{\rho}\frac{\partial}{\partial y'}\left(\mu \frac{\partial u}{\partial y'}\right) + g\beta(T - T_0) \tag{1}$$

$$\frac{\partial T}{\partial t} + V_0 \frac{\partial u}{\partial y'} = \alpha\left(\frac{\partial^2 T}{\partial y'^2} - \frac{1}{k}\frac{\partial q_r}{\partial y'}\right) \tag{2}$$

Following Carey and Mollendorf [5], the fluid dynamic viscosity (μ) is assumed to vary with temperature difference given as:

$$\mu = \mu_0(1 - \lambda\theta(y)), \ \lambda \in \Re \tag{3}$$

and the radiative heat flux q_r as given by Sparrow and Cess (1962) is:

$$q_r = \frac{-4\sigma}{3\delta}\frac{\partial T^4}{\partial y'} \tag{4}$$

with the appropriate boundary conditions for the velocity and temperature fields as:

$$u = U_0, T = T_w \ at \ y' = 0 \tag{5}$$

$$u = 0, T = T_0 \ at \ y' = h \tag{6}$$

METHOD OF SOLUTION

In order to transform the governing equations together with the initial and boundary conditions; the following similarity variables are introduced:

$$u = U_0 f(y), y = \frac{y'}{\sqrt{\upsilon t}}, \theta(y) = \frac{T - T_0}{T_w - T_0}, \delta = 2\sqrt{\upsilon t} \tag{7}$$

Using equations (3) and (7) in equation (1), the momentum equation is transformed into:

$$f''(y) = -\frac{1}{2}(y + c)f'(y)(1 + \lambda\theta(y)) + \lambda\theta'(y)f'(y)(1 + \lambda\theta(y)) - Gr\theta(y)(1 + \lambda\theta(y)), \lambda \in \Re \tag{8}$$

Similarly, using equation (4) and (7) in equation (2) gives:

$$-\frac{1}{2}y\theta'(y)(T_w - T_0) = \left(\frac{\alpha}{\upsilon t}\theta''(y)(T_w - T_0) + \frac{\alpha}{k}.\frac{\partial q_r}{\partial y'}\right) \tag{9}$$

The term $\frac{\partial q_r}{\partial y'}$ in equation (9) is simplified using Magyari and Pantokratoras (2011) as follow:

$$\frac{\partial q_r}{\partial y'} = -\frac{4\sigma}{3\delta}\frac{\partial}{\partial y'}\left[\frac{\partial T^4}{\partial \mathrm{y}'}\right] = -\frac{4\sigma}{3\upsilon t\delta}\left(\frac{\partial^2}{\partial y^2}\left(\left[\theta(y)(T_w - T_0) + T_0\right]^4\right)\right)$$

$$= -\frac{4\sigma}{3\upsilon t\delta}\left(\frac{\partial}{\partial y}\left(4\left[\theta(y)(T_w - T_0) + T_0\right]^3\right)\frac{\partial}{\partial y}\left(\theta(y)(T_w - T_0)\right)\right)$$

$$= -\frac{4\sigma}{3\upsilon t\delta}\left(12\left[\theta(y)(T_w - T_0) + T_0\right]^2\frac{\partial}{\partial y}\left(\theta(y)(T_w - T_0)\right)\frac{\partial}{\partial y}\left(\theta(y)(T_w - T_0)\right)\right)$$

$$= -\frac{4\sigma}{3\upsilon t\delta}\left(4\left[\theta(y)(T_w - T_0) + T_0\right]^3\frac{\partial^2}{\partial y^2}\left(\theta(y)(T_w - T_0)\right)\right)$$

$$= -\frac{4\sigma}{3\upsilon t\delta}\left(12(T_w - T_0)^4\left[\theta(y) + \phi\right]^2\frac{\partial}{\partial y}\left(\theta(y)\right).\frac{\partial}{\partial y}\left(\theta(y)\right)\right)$$

$$= -\frac{4\sigma}{3\upsilon t\delta}\left(4(T_w - T_0)^4\left[\theta(y) + \phi\right]^3\frac{\partial^2}{\partial y^2}\left(\theta(y)\right)\right) \tag{10}$$

Substituting equation (10) into (8) and simplifying gives:

$$\theta''(y) = -\frac{1}{2}\Pr(y + c)\theta'(y)\left[1 - \frac{4R}{3}\left[\theta(y) + \phi\right]^3\right]$$
$$-4R\left[\theta(y) + \phi\right]^2\theta'(y)\theta'(y)\left[1 - \frac{4R}{3}\left[\theta(y) + \phi\right]^3\right] \tag{11}$$

Again, using equation (6) in the boundary conditions (5) we have:

$$u = U_0,\ T = T_w \text{ at } y' = 0$$

$$\Rightarrow f \quad \Rightarrow U_0 = U_0 f(0)\ ,\ \theta(0) = 1 \text{ at } y = 0 \tag{12}$$

$$\Rightarrow 0 = U_0 f(H) \Rightarrow f(H) = 0 \; and \; \theta(H) = 0 \; at \; y = H \tag{13}$$

where $Gr = \frac{}{4\upsilon U_0}, c = \frac{}{\sqrt{\upsilon}}, \kappa = \frac{}{3k\delta}, \varphi = \frac{}{T_w - T_0}, H = \frac{h}{\sqrt{\upsilon t}}$ (14)

Mathematical Description of ADM

Consider the inhomogeneous nonlinear differential equation in Adomian's operator-theoretic form:

$$Lf + Df + Nf = q \tag{15}$$

where f is unknown function or system output, which is to be determined by a recursive relation, L is an invertible linear operator which is the highest order derivative, D is the remainder of the linear operator whose order is less than L, Nf represents the nonlinear terms and q is the system input.

Applying the inverse L^{-1} to both sides of (15) and using the given boundary conditions, we have:

$$f = u - L^{-1}(Df) - L^{-1}(Nf) \tag{16}$$

where u represents the term arising from integrating q.

The standard ADM defines the solution f by the decomposition series

$$f = \sum_{n=0}^{\infty} f_n \tag{17}$$

And the nonlinear term comprises the series of the Adomian polynomials

$$Nf = \sum_{n=0}^{\infty} A_n \tag{18}$$

where the A_n are the Adomian polynomials generated from the relation

$$A_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{\infty}\lambda_i f_i\right)\right]\right]_{\lambda=0} \tag{19}$$

such that the Adomian polynomials are evaluated as:

$$\begin{aligned} A_0 &= f(u_0) \\ A_1 &= u_1 f^{(1)}(u_0) \\ A_2 &= u_2 f^{(1)}(u_0) + \frac{1}{2!}u_1^2 f^{(2)}(u_0) \\ A_3 &= u_3 f^{(1)}(u_0) + u_1 u_2 f^{(2)}(u_0) + \frac{1}{3!}u_1^3 f^{(3)}(u_0) \\ &\ldots\ldots\ldots \\ &\ldots\ldots\ldots \end{aligned} \tag{20}$$

The solution components $f_0, f_1, f_2, \ldots\ldots\ldots$ are determined recursively as follows:

$$f_0 = u \tag{21}$$

$$f_{j+1} = -L^{-1}(Df_j) - L^{-1}(Nf_j),\ j \geq 0 \tag{22}$$

where u_0 is referred to as the zeroth-order component.

Next, we proceed to obtain the approximate solution of equations (8) and (11).

Adomian decomposition solution of the problem

Equations (8) and (11) under the boundary conditions (12) and (13) are solved using ADM as follow:

Denote by $f'' = \dfrac{d^2 f}{dy^2}$, $\theta'' = \dfrac{d^2\theta}{dy^2}$ and (23)

Let $L(y) = \dfrac{d^2 f}{dy^2}$, so that $Lf(y) = f''(y)$, $L\theta(y) = \theta''(y)$ and $L^{-1} = \int\int(.)dydy$ (24)

Using equations (23) and (24), equations (8) and (11) can be written as:

$$Lf(y) = -\frac{1}{2}(y+c)f'(y)(1+\lambda\theta(y)) + \lambda\theta'(y)f'(y)(1+\lambda\theta(y)) - Gr\theta(y)(1+\lambda\theta(y)), \lambda \in \Box \tag{25}$$

$$L\theta(y) = -\frac{1}{2}(y+c)\Pr\theta'(y)\left[1-\frac{4R}{3}[\theta(y)+\phi]^3\right]$$
$$-4R[\theta(y)+\phi]^2\theta'(y)\theta'(y)\left[1-\frac{4R}{3}[\theta(y)+\phi]^3\right] \quad (26)$$

Operating L^{-1} both sides of equations (25) and (26) we obtain:

$$L^{-1}Lf(y) = -\frac{1}{2}L^{-1}\big((y+c)f'(y)(1+\lambda\theta(y))\big)+\lambda L^{-1}\big(\theta'(y)f'(y)(1+\lambda\theta(y))\big)$$
$$-GrL^{-1}\big(\theta(y)(1+\lambda\theta(y))\big) \quad (27)$$

$$L^{-1}L\theta(y) = -\frac{1}{2}\Pr L^{-1}\left((y+c)\theta'(y)\left[1-\frac{4R}{3}[\theta(y)+\phi]^3\right]\right)$$
$$-4RL^{-1}\left([\theta(y)+\phi]^2\theta'(y)\theta'(y)\left[1-\frac{4R}{3}[\theta(y)+\phi]^3\right]\right) \quad (28)$$

By ADM, $L^{-1}Lf(y) = f(y)-f(0)-yf'(0)$ (29)

$$L^{-1}L\theta(y) = \theta(y)-\theta(0)-y\theta'(0) \quad (30)$$

Using equations (12), (29) and (30) into equations (27) and (28) we have:

$$f(y) = 1+yA-\frac{1}{2}L^{-1}\big((y+c)f'(y)(1+\lambda\theta(y))\big)+\lambda L^{-1}\big(\theta'(y)f'(y)(1+\lambda\theta(y))\big)$$
$$-GrL^{-1}\big(\theta(y)(1+\lambda\theta(y))\big) \quad (31)$$

$$\theta(y) = 1+yB-\frac{1}{2}\Pr L^{-1}\left((y+c)\theta'(y)\left[1-\frac{4R}{3}[\theta(y)+\phi]^3\right]\right)$$
$$-4RL^{-1}\left([\theta(y)+\phi]^2\theta'(y)\theta'(y)\left[1-\frac{4R}{3}[\theta(y)+\phi]^3\right]\right) \quad (32)$$

According to the standard ADM, $f(y)$ and $\theta(y)$ may be expressed as:

$$f(y) = \sum_{n=0}^{\infty} f_n(y), \theta(y) = \sum_{n=0}^{\infty}\theta_n(y) \quad (33)$$

Using equations (33) into equations (31) and (32), we have:

$$\sum_{n=0}^{\infty} f_n(y) = 1+yA-\frac{1}{2}L^{-1}\left((y+c)\frac{d}{dy}\left(\sum_{n=0}^{\infty} f_n(y)\right)\left(1+\lambda\sum_{n=0}^{\infty}\theta_n(y)\right)\right)$$
$$+\lambda L^{-1}\left(\frac{d}{dy}\left(\sum_{n=0}^{\infty}\theta_n(y)\right)\frac{d}{dy}\left(\sum_{n=0}^{\infty} f_n(y)\right)\left(1+\lambda\sum_{n=0}^{\infty}\theta_n(y)\right)\right)$$
$$-GrL^{-1}\left(\sum_{n=0}^{\infty}\theta_n(y)\left(1+\lambda\sum_{n=0}^{\infty}\theta_n(y)\right)\right) \quad (34)$$

$$\sum_{n=0}^{\infty}\theta_n(y) = 1+yB-\frac{1}{2}\Pr L^{-1}\left((y+c)\frac{d}{dy}\left(\sum_{n=0}^{\infty}\theta_n(y)\right)\left[1-\frac{4R}{3}\left[\sum_{n=0}^{\infty}\theta_n(y)+\phi\right]^3\right]\right)$$

$$-4RL^{-1}\left(\left[\sum_{n=0}^{\infty}\theta_n(y)+\phi\right]^2\frac{d}{dy}\left(\sum_{n=0}^{\infty}\theta_n(y)\right)\frac{d}{dy}\left(\sum_{n=0}^{\infty}\theta_n(y)\right)\left[1-\frac{4R}{3}\left[\sum_{n=0}^{\infty}\theta_n(y)+\phi\right]^3\right]\right) \quad (35)$$

Setting $f_0(y)=1+Ay-GrL^{-1}((\theta_0(y)(1+\lambda\theta_0(y)))$ and $\theta_0(y)=1+By$

then $f_{m+1}(y)$ and $\theta_{b+1}(y)$ for $m,b\geq 0$ are determined using the recursive relations:

$$f_{m+1}(y)=-\frac{1}{2}L^{-1}\left((y+c)\frac{d}{dy}\left(f_j(y)\right)\left(1+\lambda\theta_j(y)\right)\right)+\lambda L^{-1}\left(\frac{d}{dy}\left(\theta_j(y)\right)\frac{d}{dy}\left(f_j(y)\right)\left(1+\lambda\theta_j(y)\right)\right) \quad (36)$$

And $$\theta_{b+1}(y)=-\frac{1}{2}\Pr L^{-1}\left((y+c)\frac{d}{dy}\left(\theta_j(y)\right)\left[1-\frac{4R}{3}\left[\theta_j(y)+\phi\right]^3\right]\right)$$

$$-4RL^{-1}\left(\left[\theta_j(y)+\phi\right]^2\frac{d}{dy}\left(\theta_j(y)\right)\frac{d}{dy}\left(\theta_j(y)\right)\left[1-\frac{4R}{3}\left[\theta_j(y)+\phi\right]^3\right]\right) \quad (37)$$

Finally, the solution is given by the partial sum:

$$f(y)=\sum_{n=0}^{S}f_n \quad \text{and} \quad \theta(y)=\sum_{n=0}^{Q}\theta_n \quad (38)$$

where S and Q are truncation points such that the ADM solution converges. Convergence of ADM solution has been shown to be rapidly in Adomian (1994) and Cherruault (1990).

Using equation (38); Nusselt numbers on the plates at $y=0$ and $y=1$ are evaluated using:

$$Nu_0=-\frac{d\theta}{dy}\Big|_{y=0} \; and \; Nu_1=-\frac{d\theta}{dy}\Big|_{y=1} \quad (39)$$

and the skin friction is calculated via:

$$\tau_0=\frac{df}{dy}\Big|_{y=0} \; and \; \tau_1=\frac{df}{dy}\Big|_{y=1} \quad (40)$$

RESULTS AND DISCUSSION

Using computer algebra software package (Mathematica), equation (38) is simulated and the results are presented in Figures 2- 9 and in Tables I and II. The ambient Prandtl number is taken as 0.71 and 4 which correspond to air and R-12 refrigerant respectively. Similarly, the values of radiation, suction and viscosity variation parameters are chosen arbitrarily from 0 to 3. In addition, the value of Grashof number is taken to be 10, 12, 14. That is, $Gr>0$ corresponds to cooling of the channel by free convection current.

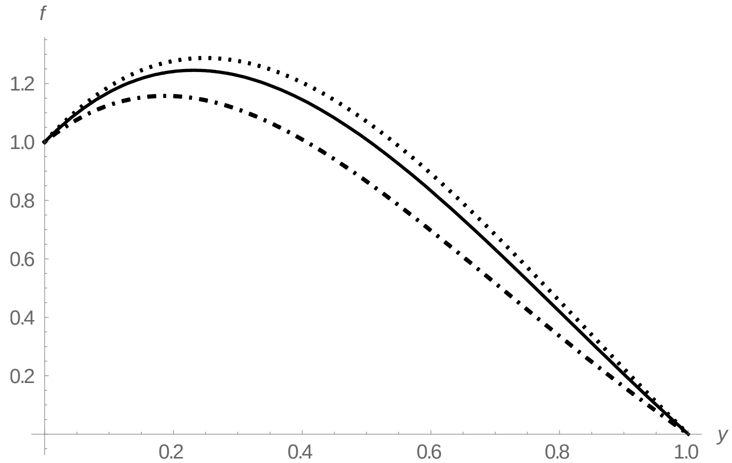

Fig. 2: Velocity profile for different Pr

(ϕ = 0.1, Pr = 0.71, $\lambda = 0.1$,R = 0.1, Gr = 10, c = 1, . . . Pr = 0.71, ____Pr = 2, . _ . _ . Pr = 4)

The effect of varying Pr is depicted in Figure 2 above. The figure reveals that the fluid velocity within the channel decreases with increase in Pr.

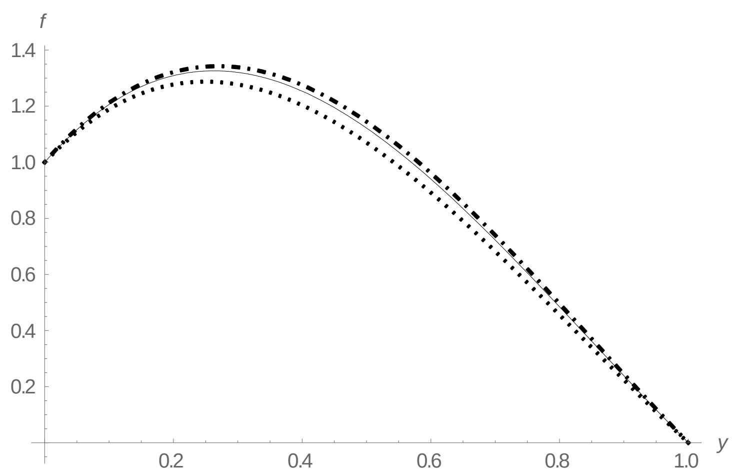

Fig. 3: Velocity profile for different R

(ϕ = 0.1, Pr = 0.71, $\lambda = 0.1$, R = 0.1, Gr = 10, c = 1, . . . R = 0.1, ____R = 0.5, . _. _. R = 1)

Figure 3 demonstrates that the velocity of the fluid within the channel increases with increase in radiation parameter. This is due to the fact that, when R increases, it amounts to increasing the buoyancy force of the fluid molecules within the channel.

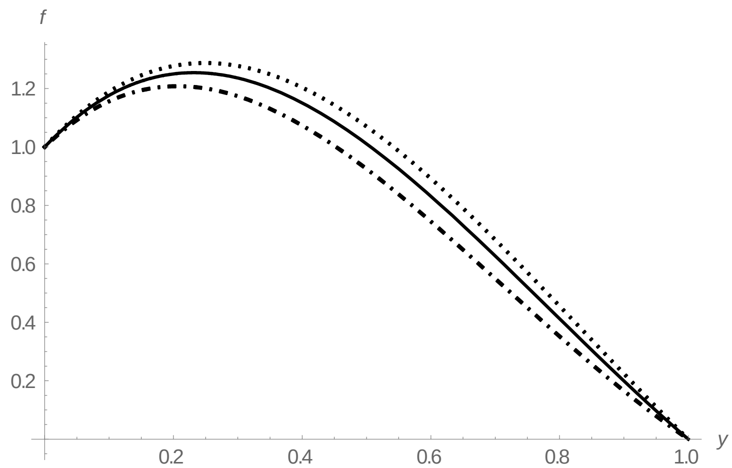

Fig. 4: Velocity profile for different c

(ϕ=0.1, Pr = 0.71, R = 0.1, λ = 0.1, Gr = 10, - . -. - . c = 1, ____c = 0.5, ... c = 0.1)

Figure 4 shows that; the fluid velocity within the channel decreases with increase in suction parameter. This is physically true that; when fluid is in motion, suction of its portion from the bounding surface causes a distortion in the fluid velocity near the suction area.

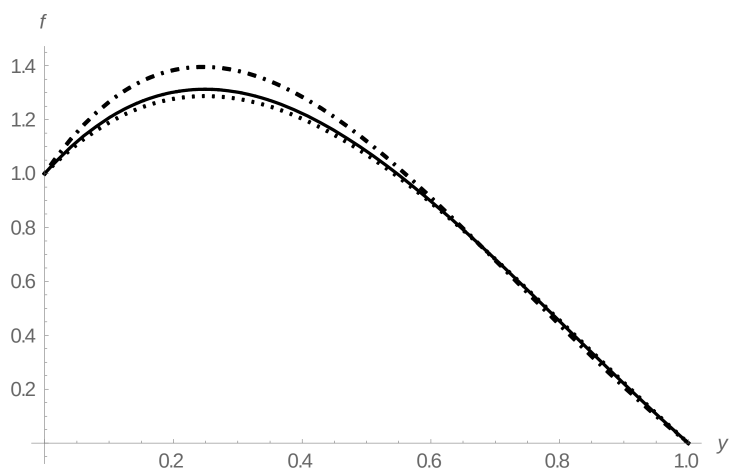

Fig. 5: Velocity profiles for different λ

(ϕ = 0.1, R = 0.1, Gr = 10, c = 0.1, Pr = 0.71, ..., λ = 0.1, ____λ = 0. 2, . _. _. λ = 0.6)

Figure 5 depicts that, the fluid velocity increases with increase in λ. This is attributed to the fact that, when λ increases it results to the lessening of the fluid viscosity and this consequently increase the fluid velocity.

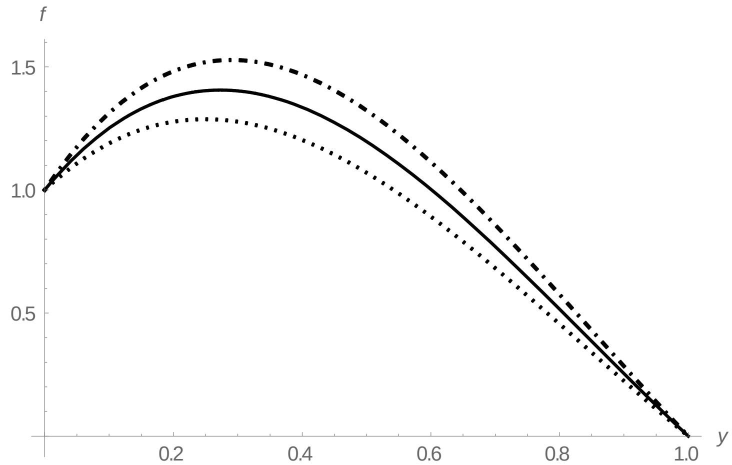

Fig. 6: velocity profile for different Gr

(R = 0.1, ϕ = 0.1, Pr = 0.71, $\lambda = 0.1$, c = 0.1, . . .Gr = 10, ___Gr = 12, - . - . – Gr = 14)

The effect of varying Gr is pictured in figure 6. The figure depicts that; the fluid velocity increases with increase in Gr. This is attributed to the fact that; an increase in Gr implies a corresponding increase in the buoyant force of the fluid molecules within the channel, hence an increase in the speed of the fluid during the flow.

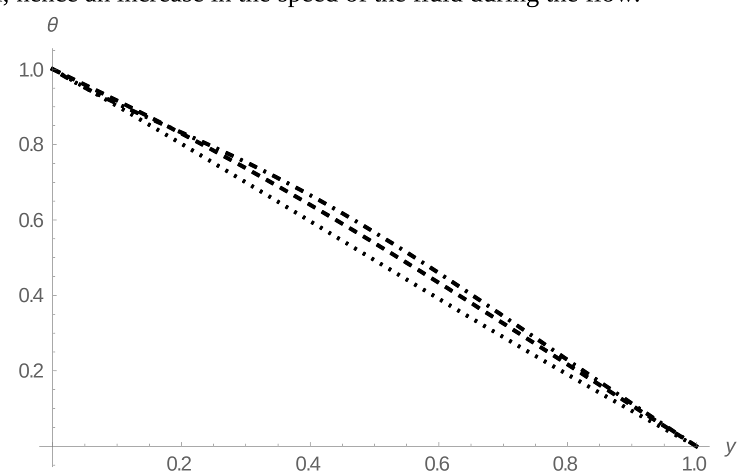

Fig. 7: Temperature profiles for different R

(c = 0.1, Pr = 0.71, ϕ = 0.1, . . .R = 0.1, - - - R = 0.5, . - . -R = 1)

The effect of radiation parameter (R) is reflected in figure 6. It is viewed that, the fluid temperature increases with increase in R. This is due to the fact that, when R increases it result to decrease in thermal conductivity of the fluid within the channel.

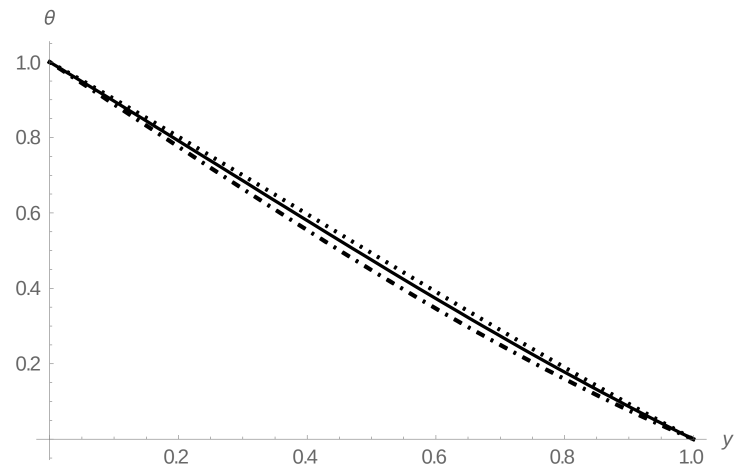

Fig. 8: Temperature profiles for different c

(R=0.1, Pr=0.71, ϕ = 0.1, ... c = 1, ____c = 0.5, - . - . c = 0.1)

Figure 8 above depicts that a decrease in fluid suction (c) results to the increase in the fluid temperature within the channel. This trend is as a result of concentration of the fluid molecules near the suction area; this transitively leads to the increase in temperature due to collision of the fluid molecules.

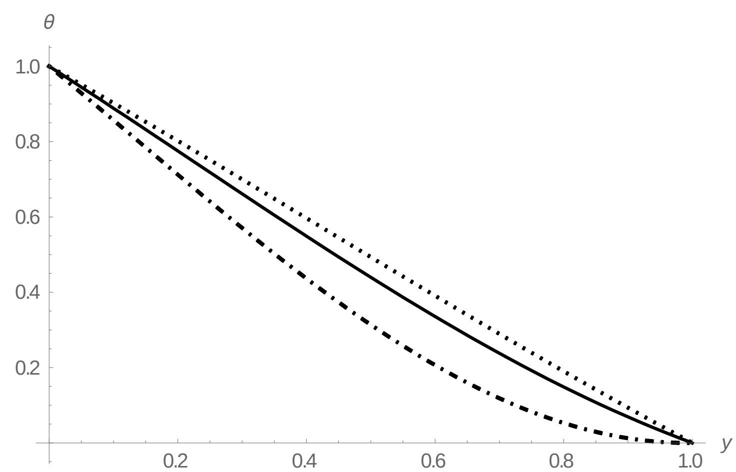

Fig. 9: Temperature profile for different Pr

(R = 0.1, c = 0.1, ϕ = 0.1, ... Pr = 0.71, _____ Pr = 2, - . - . Pr = 4)

From figure 9, it is observed that the temperature of the fluid decreases with increase in Pr. Prandtl number (Pr) signifies the ratio of momentum diffusivity to thermal conductivity of fluids. Fluids with higher Prandtl number possess lower thermal conductivity as such heat diffused faster in fluid with lower Prandtl than in fluid with higher Pr.

Table I: Numerical values of skin friction on the walls

λ	R= 0.1, Ø= 0.21, Pr =0.71, Gr = 10, c = 0.001		R = 0.2,Ø= 0.21, Pr =0.71, Gr = 10, c = 0.001		R = 0.2,Ø = 0.5, Pr = 0.71, Gr = 10, c = 0.001	
	τ_0	τ_1	τ_0	τ_1	τ_0	τ_1
0.1	2.49716	2.34804	2.56299	2.40398	2.61158	2.45774
0.2	2.74505	2.40670	2.81760	2.46710	2.87109	2.52454
0.3	2.99297	2.46461	3.07220	2.52946	3.13060	2.59056
0.4	3.24091	2.52177	3.32679	2.59106	3.39012	2.65580
0.5	3.48889	2.57819	3.58138	2.65191	3.64963	2.72027

Table I shows the effects of varying viscosity parameter (λ) on the skin friction. It reveals that the skin friction between the channel walls and the working fluid increases on all the walls with increase in λ . Similarly, the skin friction is seen to increase with increase in R and Ø for some fixed values of Pr and Ø.

Table II: Numerical values for the rate of heat transfer on the channel walls

Pr	R=0.1,Ø=0.21, c=0.001		R=0.2,Ø=0.21, c=0.001		R=0.2,Ø=0.5, c=0.001	
	Nu_0	Nu_1	Nu_0	Nu_1	Nu_0	Nu_1
0.44	0.908144	0.998224	0.844077	1.053570	0.841484	1.127100
0.71	0.924742	0.956932	0.856568	1.017510	0.850279	1.097740
1.00	0.943317	0.910685	0.870481	0.977304	0.860029	1.065170
2.00	1.014080	0.734268	0.922855	0.825604	0.896311	0.943703
3.00	1.097430	0.526381	0.922855	0.825604	0.937418	0.805717
4.00	1.197150	0.278266	1.054080	0.444814	0.984510	0.647312

The effect of Prandtl number on the rate of heat transfer between the walls and the working fluid is displayed in Table II. The table displayed that; the Nusselt number on the wall at $y = 0$ increases with increase in Pr while it is seen to decrease with increase in Pr on the wall stationed at at $y = 1$. Furthermore, the Nusselt number on the wall kept at $y = 0$ is observed to decrease with increase in R whereas it is noticed to increase with increase in R on the wall at $y = 1$. Again, the Nusselt number on the wall positioned at $y = 0$ is found to decrease with increase in Ø while it is viewed to increase on the wall at $y = 1$ with increase in Ø.

CONCLUSION

The paper investigates the combined effects of variable viscosity and thermal radiation on unsteady natural convection flow through a vertical porous channel and the results are presented and discussed. Our investigation shows that; the fluid velocity increases with increase in λ. It is worthy to mention here that when λ and R tend to zero; the results obtained herein coincide with that of Makinde *et al.* (2007). Moreover, it can be concluded that, when the viscosity of any working fluid is sensitive to temperature change; the effect of variable viscosity has to be taken into consideration. This study is hoped to serve as a complement to previous studies and also be an avenue for further researches.

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Nomenclature and Greek symbols

Symbols	Interpretation	Unit
y'	dimensional length	m
y	dimensionless length	
t	time	s
g	gravitational acceleration	ms^{-2}
k	thermal conductivity	W/mK
δ	absorption coefficient	
T	dimensional temperature of the fluid	K
h	dimensional channel width	m
T_w	wall temperature	K
T_0	ambient temperature	K
V	velocity of suction	ms^{-1}
u	dimensional velocity	
v	kinematic viscosity of the fluid	m^2s^{-1}
α	thermal diffusivity of the fluid	
β	volumetric expansion coefficient	K^{-1}
μ	variable viscosity	$kgm^{-1}s^{-1}$
q_r	radiative heat flux	Wm^{-2}
θ	dimensionless temperature	
U_0	reference velocity	ms^{-1}
λ	viscosity variation parameter	K^{-1}
$\emptyset$	temperature difference parameter	K
R	radiation parameter	
$\Re$	set of real numbers	
Gr	Grashof number	
Pr	Prandtl number	
μ_0	reference fluid viscosity	$kgm^{-1}s^{-1}$
c	suction parameter	
H	dimensionless channel width	
Nu_0	Nusselt number at the plate $y = 0$	
Nu_1	Nusselt number at the plate $y = 1$	
τ_0	skin friction on the plate at $y = 0$	
τ_1	skin friction on the plate at $y = 1$	

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