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UNSTEADY NATURAL CONVECTION FLOW THROUGH A VERTICAL POROUS CHANNEL FILLED WITH POROUS MATERIALS UNDER THE EFFECT OF THERMAL RADIATION

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ABSTRACT:
The paper examines the velocity and heat transfer on unsteady natural convection flow through a vertical porous channel filled with porous materials under the effect of thermal radiation. The study considered the fluid flow to be through an infinite vertical porous channel filled with porous materials and the energy equation in the flow model is examined using non-linear Rosseland heat diffusion. The partial differential equations associated with the flow formation are transformed into ordinary differential equations (ODEs) using similarity variables and the resulting ODEs are solved by Adomian decomposition method (ADM). Finally, the results are presented on graphs as velocity and temperature profiles for various values of the controlling physical parameters involved in the problem. In the course of investigation, it was found that; the fluid velocity increases with increase in thermal radiation and Darcy number and the fluid temperature was seen to increase with increase in thermal radiation.

Keywords: *Natural convection; Thermal radiation; Porous Materials; ADM.*

INTRODUCTION:

One of the basic flows in fluid dynamics is natural convection flow where the fluid motion is induced by density difference occurring between the fluid particles due to temperature gradients. In this mechanism, the fluid surrounding the heat source receives heat, becomes less dense and raise, the surrounding cooler fluid then moves to replace it. Natural convection flow occur in many technological and industrial applications such as in nuclear reactors, rapid cooling process, motion of fluid in computer equipment, radiators, storage devices, cooling of electronic equipment inside computers, furnaces etc. In view of the applications of free convective heat transfer flows through porous medium, comprehensive studies has been made by Cheng (1978), Ingham and Pop (1998), Neild and Bejan (2013) and Vafai (2000). The use of porous medium allows suction/injection of fluid through the bounding surface and the significance of suction/injection is well recognized in Singh (1984). Ishak (2008) lamented that suction or injection of fluid through bounding surface can significantly change the flow field and this consequently affect the heat transfer rate from the porous plate. Al-Sanea (2004) disclosed that suction tends to increase skin friction and heat transfer coefficient whereas injection acts in the opposite manner. This is well explored by Shojaefard *et al.* (2005) where suction and injection were used to control fluid flow on the surface of a subsonic aircraft. Braslow (1999) stated that by controlling the flow as such, fuel consumption might be decreased by 30%, a considerable reduction in pollutant emission is achieved. In a related literature, the study of natural convection flow through vertical porous stratum and transient convective flow in a vertical channel with heat sink respectively were examined in Jha (1997 & 2003).

Thermal radiation refers to the emission of heat due to entropy generation by systems, when it occurs; the consequent effect is poor performance of systems. For minimization of entropy generation for maximum output of systems and also for safety of lives and properties, thermal radiation by systems needs to be controlled. In view thereof several studies were accorded to this subject, this can be seen in Hossain and Takhar (1996), Hossain *et al.* (1999a), Hossain *et al.* (2001b), Rapist (2004), Cortell (2007a &

2008b), Makinde and Ogulu (2011), Makinde *et al.* (2007) and Ajibade (2010), a few to mention among others. Sparrow and Cess (1978) gave the expression for radiative heat flux which is widely used in the studies of heat transfer in boundary layer flows. Several researchers such as have used a linearized form of temperature on this expression. This was however, condemned by Magyari and Pantokratoras (2011) on the basis that it does not reflects the real characteristics of heat emission or conduction as such they proposed alternative approach where the energy equation is evaluated using non-linear Rossland heat diffusion. Despite series of investigation conducted on fluid flows with thermal radiation; only few researchers such as Abdul-Hakeem and Sathiyanathan (2009), Yabo *et al.* (2016), and Jha *et al.* (2017) and Mansour (1990) have adopted the new technique. The present paper studied unsteady natural convection flow through a vertical porous channel with thermal radiation effect using the method of ADM (Adomian (1994)) and non-linear Rossland heat diffusion.

MATHEMATICAL PROBLEM:

Figure 1 below shows a schematic diagram of unsteady natural convection flow of an incompressible viscous fluid through an infinite vertical porous channel filled with porous materials under the effect of thermal radiation. The channel plates are stationed h distance apart in the presence of an incidence radiative heat flux of intensity q_r which is absorbed by the plates and transferred to the fluid**.** Since the plates are of infinite length, the flow direction is assumed vertically upward and the fluid is considered optically thick where the radiative heat flux of Rosseland diffusion approximation can be utilized. The x'-axis is taken along the channel in the vertically upwards direction, being the direction of the flow while the y'-axis is taken normal to it. In addition, the effect of radiative heat flux in the x'-direction is considered negligible compared to that in the y'-direction. At time $t \le 0$, both the fluid and the plates are given a reference velocity U_0 with constant temperature T_0. At time $t > 0$, the temperature of the plate kept at $y' = 0$ rise to T_w while the other plate at $y' = h$ is fixed and maintained at temperature T_0.

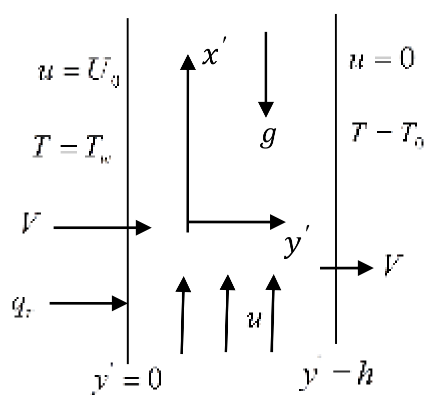


Figure 1: Schematic diagram of the problem

Under these assumptions, the appropriate governing equations are:

$$\frac{\partial u}{\partial t} + V\frac{\partial u}{\partial y'} = \gamma_{eff}\frac{\partial^2 u}{\partial y'^2} + g\beta(T - T_0) - \frac{\gamma u}{K} \quad (1)$$

$$\frac{\partial T}{\partial t} + V\frac{\partial T}{\partial y'} = \alpha\frac{\partial^2 T}{\partial y'^2} + \frac{\alpha}{k}\frac{\partial q_r}{\partial y'} \quad (2)$$

where q_r is the radiative heat flux given by Sparrow and Cess [22] as:

$$q_r = \frac{-4\sigma}{3\delta}\frac{\partial T^4}{\partial y^{'}} \tag{3}$$

with the boundary conditions:

$$u = U_0, T = T_w \; at \; y^{'} = 0 \tag{4}$$

$$u = 0, T = T_0 \; at \; y^{'} = h \tag{5}$$

3.0 Method of Solution:

In order to transform the governing equations into dimensionless form; the following similarity variables are utilized:

$$u = U_0 f(y), \theta(y) = \frac{T - T_0}{T_w - T_0}, \delta = 2\sqrt{\gamma t} \tag{6}$$

Using equations (3) and (6) in equation (1)-(2), we obtained the following differential equations:

$$f^{''}(y) = \frac{-1}{2\Gamma}(y+c)f^{'}(y) - \frac{Gr\theta(y)}{\Gamma} + \frac{f(y)}{\Gamma Da} \tag{7}$$

$$-\frac{1}{2}(y+c)\theta^{'}(y)(T_w - T_0) = \left(\frac{\alpha}{\upsilon t}\theta^{''}(y)(T_w - T_0) - \frac{\alpha}{k\sqrt{\upsilon t}}\frac{\partial q_r}{\partial y}\right) \tag{8}$$

Following Magyari and Pantokratoras (2011); the term $\frac{\partial q_r}{\partial y^{'}}$ is simplified as follows:

$$\frac{\partial q_r}{\partial y^{'}} = -\frac{4\sigma}{3\delta}\frac{\partial}{\partial y}\left[\frac{\partial T^4}{\sqrt{\upsilon t}\partial \mathrm{y}}\right]$$

$$= -\frac{4\sigma}{3\delta\sqrt{\upsilon t}}\left(\frac{\partial^2}{\partial y^2}\left(\left[\theta(y)(T_w - T_0) + T_0\right]^4\right)\right)$$

$$= -\frac{4\sigma}{3\delta\sqrt{\upsilon t}}\left(\frac{\partial}{\partial y}\left(4\left[\theta(y)(T_w - T_0) + T_0\right]^3\right)\frac{\partial}{\partial y}\left(\theta(y)(T_w - T_0)\right)\right)$$

$$= -\frac{4\sigma}{3\delta\sqrt{\upsilon t}}\left(12\left[\theta(y)(T_w - T_0) + T_0\right]^2\frac{\partial}{\partial y}\left(\theta(y)(T_w - T_0)\right)\frac{\partial}{\partial y}\left(\theta(y)(T_w - T_0)\right)\right)$$

$$-\frac{4\sigma}{3\delta\sqrt{\upsilon t}}\left(4\left[\theta(y)(T_w - T_0) + T_0\right]^3\frac{\partial^2}{\partial y^2}\left(\theta(y)(T_w - T_0)\right)\right)$$

$$= -\frac{4\sigma}{3\delta\sqrt{\upsilon t}}\left(12(T_w - T_0)^4\left[\theta(y) + \phi\right]^2\frac{\partial}{\partial y}\left(\theta(y)\right).\frac{\partial}{\partial y}\left(\theta(y)\right)\right)$$

$$-\frac{4\sigma}{3\delta\sqrt{\upsilon t}}-\frac{4\sigma}{3\delta\sqrt{\upsilon t}}\left(4(T_w-T_0)^4\left[\theta(y)+\phi\right]^3\frac{\partial^2}{\partial y^2}\left(\theta(y)\right)\right) \quad (9)$$

Substituting equation (9) in equation (8) gives:

$$\theta''(y)=\frac{-1}{2}\Pr(y+c)\theta'(y)\left[1-\frac{4R}{3}\left[\theta(y)+\phi\right]^3\right]-4R\left[\theta(y)+\phi\right]^2\theta'(y)\theta'(y)\left[1-\frac{4R}{3}\left[\theta(y)+\phi\right]^3\right] \quad (10)$$

furthermore, using equations (3) and (6) in equation (4) and (5), the boundary conditions are now:

$$f(0)=\theta(0)=1 \quad (11)$$

$$f(H)=\theta(H)=0 \quad (12)$$

Where $\Gamma=\frac{\gamma_{eff}}{\gamma}, Gr=\frac{4\sigma(T_w-T_0)^3}{3k\delta}, \phi=\frac{T_0}{T_w-T_0}, H=\frac{h}{\sqrt{\gamma t}}, Da=\frac{k}{\gamma t}$

$$R=\frac{4\sigma(T_w-T_0)}{3k\delta}, c=\frac{-Vt^{\frac{1}{2}}}{\sqrt{\gamma}}, \Pr=\frac{\mu c_p}{k} \quad (13)$$

Adomian decomposition solution of the problem:

For details on ADM refer to Adomian (1994).

Equations (7) and (10) under the boundary conditions (11) and (12) are solved using ADM as follows:

Denote by $f''=\frac{\partial^2 f}{\partial y^2}, \theta''=\frac{\partial^2\theta}{\partial y^2}$ so that $Lf=f'', L\theta=\theta''$ *and* $L^{-1}=\int_0^y\int_0^y(.)dYdY$ (14)

Using equations (14), equations (7) and (10) can be written as:

$$Lf=\frac{-1}{2\Gamma}(y+c)f'(y)-\frac{Gr\theta(y)}{\Gamma}+\frac{f(y)}{\Gamma Da} \quad (15)$$

$$L\theta(y)=-\frac{1}{2}\Pr(y+c)\theta'(y)\left[1-\frac{4}{3}R[\theta(y)+\phi]^3\right]$$

$$-4R[\theta(y)+\phi]^2\theta'(y)\theta'(y)\left[1-\frac{4}{3}R[\theta(y)+\phi]^3\right] \quad (16)$$

Operating L^{-1} on both sides of equations (15) and (16) we obtain:

$$L^{-1}Lf = \frac{-1}{2\Gamma}L^{-1}((y+c)f^{'}(y)) - \frac{Gr}{\Gamma}L^{-1}(\theta(y)) + \frac{1}{\Gamma Da}L^{-1}(f(y)) \quad (17)$$

$$L^{-1}L\theta(y) = -\frac{1}{2}\Pr L^{-1}\left((y+c)\theta^{'}(y)\left[1-\frac{4}{3}R[\theta(y)+\phi]^{3}\right]\right)$$

$$-4RL^{-1}\left([\theta(y)+\phi]^{2}\theta^{'}(y)\theta^{'}(y)\left[1-\frac{4}{3}R[\theta(y)+\phi]^{3}\right]\right) \quad (18)$$

By ADM, $L^{-1}Lf(y) = f(y) - f(0) - yf^{'}(0)$ (19)

$$L^{-1}L\theta(y) = \theta(y) - \theta(0) - y\theta^{'}(0) \quad (20)$$

Using equations (11), (19) and (20) in equations (17) and (18) we have:

$$f(y) = 1 + yA - \frac{1}{2\Gamma}L^{-1}\left((y+c)f^{'}(y)\right) - \frac{1}{\Gamma}GrL^{-1}(\theta(y)) - \frac{1}{\Gamma Da}L^{-1}(f(y)) \quad (21)$$

$$\theta(y) = 1 + yB - \frac{1}{2}\Pr L^{-1}\left((y+c)\theta^{'}(y)\left[1-\frac{4}{3}R[\theta(y)+\phi]^{3}\right]\right)$$

$$-4RL^{-1}\left([\theta(y)+\phi]^{2}\theta^{'}(y)\theta^{'}(y)\left[1-\frac{4}{3}R[\theta(y)+\phi]^{3}\right]\right) \quad (22)$$

Where $A = f^{'}(0),\ B = \theta^{'}(0)$ are assumed values to be determined based on the boundary

condition at H (23)

According to the standard ADM, $f(y)$ and $\theta(y)$ may be expressed as:

$$f(y) = \sum_{n=0}^{\infty} f_n(y),\ \theta(y) = \sum_{n=0}^{\infty}\theta_n(y), \quad (24)$$

using equation (30) into equations (27) and (28), we have:

$$\sum_{n=0}^{\infty} f_n(y) = 1 + yA - \frac{1}{\Gamma}L^{-1}\left((y+c)\frac{d}{dy}\left(\sum_{n=0}^{\infty} f_n(y)\right)\right) - \frac{1}{2\Gamma}GrL^{-1}\left(\sum_{n=0}^{\infty}\theta_n(y)\right)$$

$$-\frac{1}{\Gamma Da}L^{-1}\left(\sum_{n=0}^{\infty} f_n(y)\right) \quad (25)$$

$$\sum_{n=0}^{\infty}\theta_n(y)=1+yB-\frac{1}{2}\Pr L^{-1}\left((y+c)\frac{d}{dy}\left(\sum_{n=0}^{\infty}\theta_n(y)\right)\left[1-\frac{4}{3}R\left[\sum_{n=0}^{\infty}\theta_n(y)+\phi\right]^3\right]\right)$$

$$-4RL^{-1}\left(\left[\sum_{n=0}^{\infty}\theta_n(y)+\phi\right]^2\frac{d}{dy}\left(\sum_{n=0}^{\infty}\theta_n(y)\right)\frac{d}{dy}\left(\sum_{n=0}^{\infty}\theta_n(y)\right)\left[1-\frac{4}{3}R\left[\sum_{n=0}^{\infty}\theta_k(y)+\phi\right]^3\right]\right) \quad (26)$$

Setting $f_0(y)=Ay$ and $\theta_0(y)=1+By$ then $f_{k+1}(y)$ and $\theta_{k+1}(y)$ for $k\geq 0$ are determined using the recursive relations:

$$f_{k+1}(y)=1+yA-\frac{1}{2\Gamma}L^{-1}\left((y+c)\frac{d}{dy}(f_k(y))\right)-\frac{1}{2\Gamma}GrL^{-1}(\theta_k(y))$$

$$-\frac{1}{\Gamma Da}L^{-1}(f_k(y)),\ k\geq 0 \quad (27)$$

$$\theta_{k+1}(y)=1+yB-\frac{1}{2}\Pr L^{-1}\left((y+c)\frac{d}{dy}(\theta_k(y))\left[1-\frac{4}{3}R[\theta_k(y)+\phi]^3\right]\right)$$

$$-4RL^{-1}\left([\theta_k(y)+\phi]^2\frac{d}{dy}(\theta_k(y))\frac{d}{dy}(\theta_k(y))\left[1-\frac{4}{3}R[\theta_k(y)+\phi]^3\right]\right),k\geq 0 \quad (28)$$

Finally, the solution is given by the partial sum:

$$f(y)=\sum_{n=0}^{S}f_n(y),\ \theta(y)=\sum_{n=0}^{Q}\theta_n(y), \quad (29)$$

where S *and* Q are the truncation points for which the ADM solution converges.

CONVERGENCE OF THE ADM SOLUTION AND TERMINATION CRITERION:

ADM solution has been proved to be rapidly convergent in Adomian (1994) and Cherruault (2010). Nevertheless, to justify the convergence of our ADM solution in the present problem; we deploy the method of ratio test. Using computer algebra package "MAPLE", the following terms were obtained:

$\theta_0=0.526624, \theta_1=-0.028944, \theta_2=0.000451111, \theta_3=-0.000005611, \theta_4=5.530332075\times 10^{-12}$,

f_0 =0.1662534314, f_1 =- 0.003462435996, f_2 = 0.00006489989562, f_3 =-0.655893305$\times 10^{-7}$,

f_4 =1.173322393$\times 10^{-8}$. On the application of ratio test; we have:

$$\left|\frac{\theta_1}{\theta_0}\right| = 0.05496141460, \left|\frac{\theta_2}{\theta_1}\right| = 0.01558561360, \left|\frac{\theta_3}{\theta_2}\right| = 0.01243599122, \left|\frac{\theta_4}{\theta_3}\right| = 0.009857989438,$$

$$\left|\frac{f_1}{f_0}\right| = 0.02082625283, \left|\frac{f_2}{f_1}\right| = 0.1874399865, \left|\frac{f_3}{f_2}\right| = 0.001487813380, \left|\frac{f_4}{f_3}\right| = 0.01215136038$$

It can be seen from the above computations that; the ratio test has been fulfilled since

$$\lim_{j\to\infty}\left|\frac{f_{j+1}}{f_j}\right| < 1, \text{ for } j \geq 0$$ Robert (2010).

Hence the ADM solution of the problem converges.

For a series solution to be meaningful, the series needs to be terminated at a point such that the contribution of any additional term is negligible to the final solution. As such a termination criterion is used such that we truncate the series whenever $|f_i| < \varepsilon$ and $|\theta_i| < \delta$. For the present problem, we have chosen $\varepsilon = 5*10^{-5}$ and $\delta = 5*10^{-6}$. Considering this assumption, the solution is thus truncated after the 3rd term and the approximate solution is:

Nusselt number on the plates bounding the channel is evaluated using the formulas:

$$Nu_0 = \frac{-d\theta}{dy}\Big|_{y=0} \text{ and } Nu_1 = -\frac{d\theta}{dy}\Big|_{y=1} \quad (36)$$

and the skin friction is calculated via:

$$\tau_0 = \frac{df}{dy}\Big|_{y=0} \text{ and } \tau_1 = \frac{df}{dy}\Big|_{y=1}. \quad (37)$$

RESULTS AND DISCUSSION:

Using computer algebra software package (MAPLE), the equations in (29) were iterated and the result are presented graphically as velocity and temperature profiles. For the purpose of discussion, the value of Pr has been chosen as 0.015 and 0.71 which correspond to mercury and air respectively. The values of radiation parameter, temperature difference parameter, Darcy number and suction parameter are chosen arbitrarily between 0.1 and 5 while that of Gr has been chosen as 10, 12 and 14 i.e. Gr > 0 which correspond to cooling of the channel by free convection current.

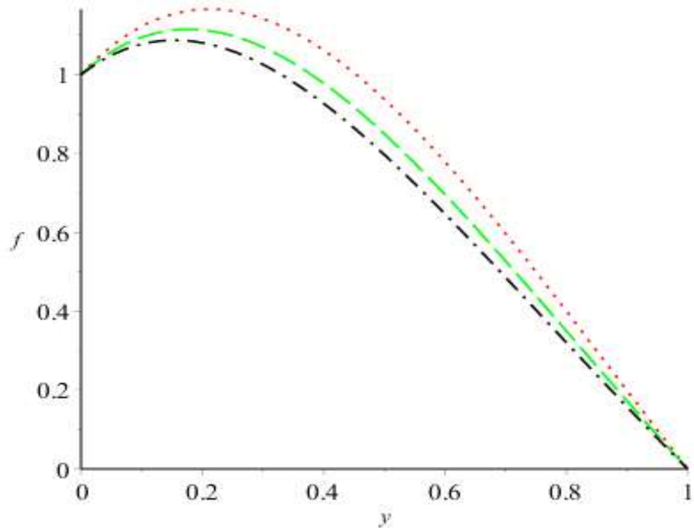


figure 2: velocity profile for different Da
(c=0.1 R=0.1 Pr=0.71, Gr=10, Γ=0.1, ...Da=0.1, ___ __Da=0.2,
- . - . -Da=0.3)

Figure 2 shows the velocity distribution for different values Da. It can be seen from the figure that, an increase in Da results into a corresponding increase in the fluid velocity. This is evidently due to the fact that as Da increases it implies an increase in the size of the pore spaces as such it results to the increase in the fluid velocity within the channel.

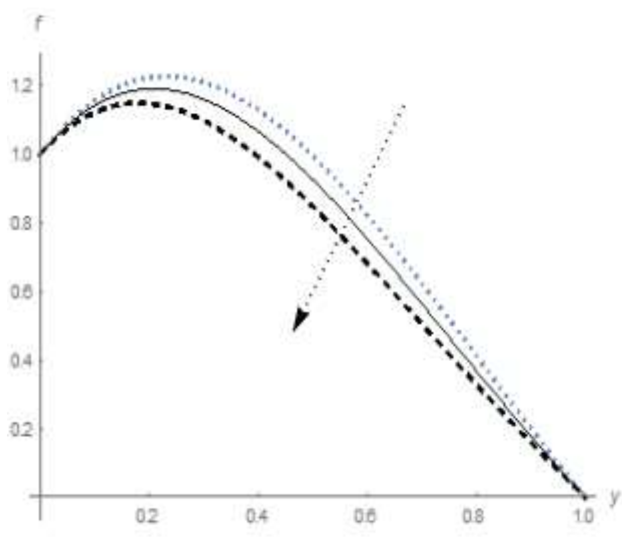


Figure 3: velocity profile for
R = 0.1, Da =1, Gr = 10, Pr = 0.71, = 1, c = { 1, 3, 5}

The effect of suction parameter (c) is shown in figure 3. It is observed that the velocity of the fluid decreases with increase in c. This is physically true that, when fluid is in motion, suction tends to slow down its velocity.

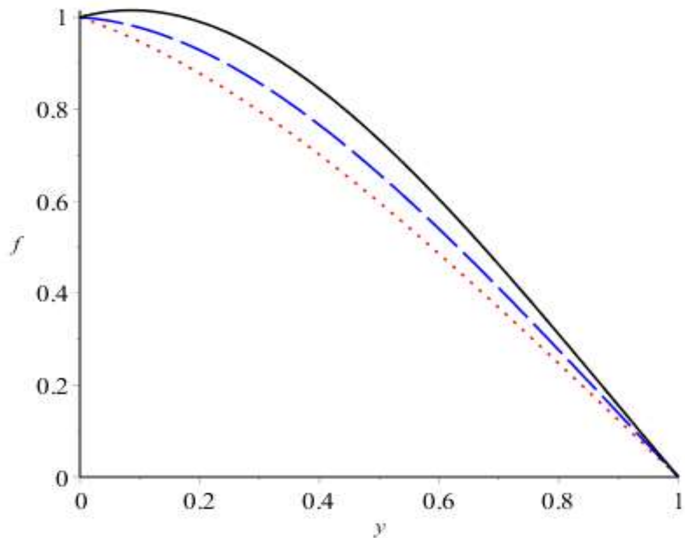
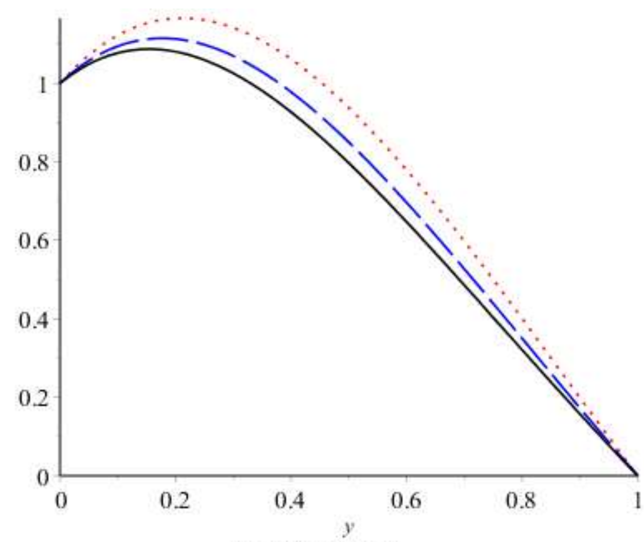


figure 4: velocity profile for different Γ
(c=0.1 R=0.1 Da=0.1, Gr=10, Pr=0.71, ...Γ=5, __ __ __Γ=3, ______Γ=2)

Figure 4 depicts the effects of Γ on the fluid velocity. The graph shows that, the fluid velocity within the channel increases with increase in Γ. The increase is Γ is caused as a results of increase in the kinematic viscosity of the fluid (γ) within the channel.



Velocity profile
c=0.1 R=0.1 Da=1, Gr=10, Γ=1, ...Pr=0.71, __ __ __Pr=3.71, ______Pr=5.71

Figure: 5

The effect of Pr is reflected in figure 5. It is observed that, the velocity of the fluid within the channel decreases with increase in Pr. This is attributed to the fact that, when Pr increases it leads to the weakening of convection current of the fluid molecules which is caused by decrease in temperature.

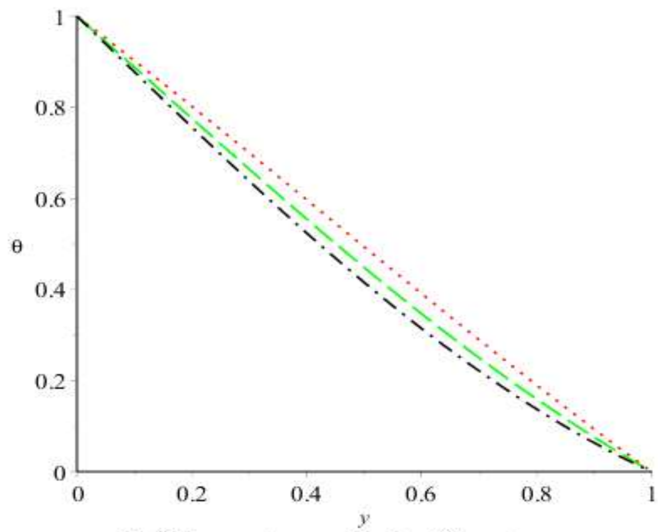


fig 6: Temperature profile for different c
(R=0.1,Pr=0.71,ϕ=0.1…….c=0.1,- - - -c=1, c=1.5

The effect of suction parameter (c) on the fluid temperature is exposed in figure 6. It shows that the fluid temperature decreases with increase in c within the channel. Suction of fluid from bounding surface results into a decrease in the fluid temperature.

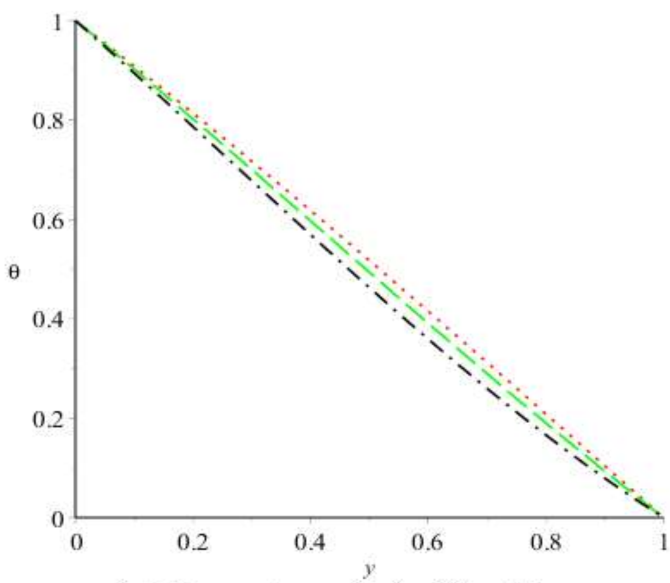


fig 7: Temperature profile for different Pr
(c=0.1,R=0.1,ϕ=0.1…….Pr=0.015,- - - -Pr=0.71, . - . - Pr=1.5

Figure 7 shows the effect of Prandtl number (Pr) on the fluid temperature. It presents an inverse relationship between the fluid temperature and the Prandtl number i.e. when Pr increases, it amounts to a corresponding decrease in thermal diffusion by the fluid within the channel thereby causing a decrease in heat flux.

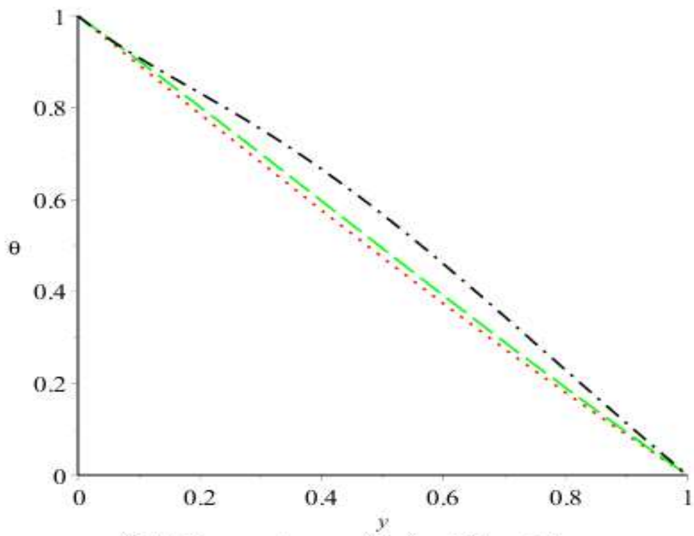


fig 8: Temperature profile for different R
(c=0.1,Pr=0.71,ϕ=0.1.......R=0.1,- - - -R=0.5, . - . - R=1

The effect of radiation parameter (R) is depicted in figure 8; it is observed that, the fluid temperature increases with increase in radiation parameter (R). Thus, increasing R result into high temperature generation within the channel.

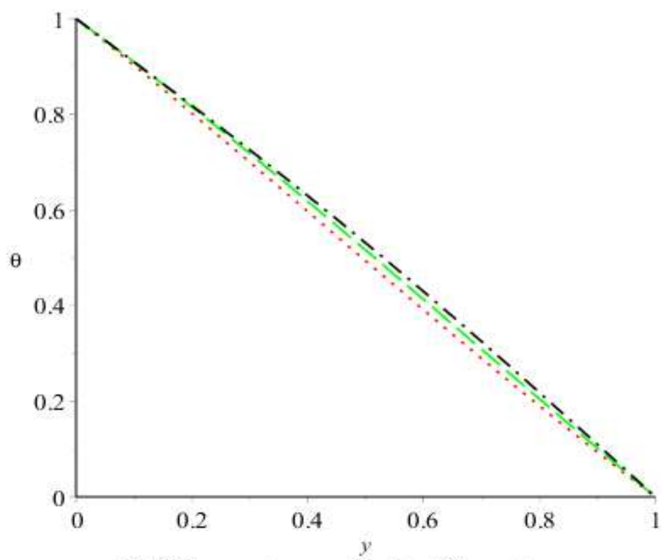


fig 9: Temperature profile for different ϕ
(c=0.1,R=0.1,Pr=0.71.......ϕ=0.1,- - - -ϕ=0.5, . - . - ϕ=1

The effect of temperature difference parameter (ϕ) on the fluid temperature is illustrated in figure 9. It is observed that, decreasing the value of $\left(T_w - T_0\right)$ leads to increasing ϕ and this consequently results into a corresponding increase in the fluid temperature.

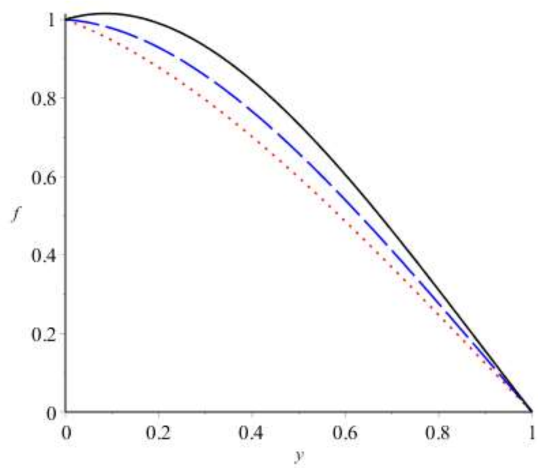


fig 10 : velocity profile for different R
(c=0.1 Γ=0.1 Da=1, Gr=10, Pr=0.71, ...R=0.1, __ __ __R=0.3, ______R=0.6

In figure 10, the effect of R on the velocity is depicted. The figure shows that; the fluid velocity increase with increase in thermal radiation. This is due to the fact that, increasing the value of R results into a corresponding increase in the rate of thermal radiation emission and the resulting effect is the rapid flow of the fluid.

Table I: Numerical values of skin friction on the walls

c	R=0.1, ∅=0.1,Pr=0.71, Gr=10, $\Gamma = 0.1$, Da=0.002		R=0.3, ∅=0.1,Pr=0.71, Gr=10, $\Gamma = 0.1$, Da=0.002	
	τ_0	τ_1	τ_0	τ_1
0.1	6.984994460	8.545898172	7.180914488	8.779065584
0.2	6.941262847	8.504517853	7.146356727	8.747912806
0.3	6.896583075	8.462251836	7.111277757	8.716356302
0.4	6.850909935	8.419049014	7.075658850	8.684374994
0.5	6.804195079	8.374854719	7.039480283	8.651946565

Table I shows the effects of varying suction parameter (c) on the skin friction. The table revealed that, the skin frictions between the channel walls and the working fluid decreases on both the plates with increase in c i.e. suction reduces skin friction on the plates. Furthermore, the skin friction on the channel walls are also seen to decrease with increase in R.

Table II: Numerical values for the rate of heat transfer on the channel walls

c	R=0.1, Ø=0.1, c=0.1, Pr=0.71		R=0.3, Ø=0.1, Pr=0.71	
	Nu_0	Nu_1	Nu_0	Nu_1
0.1	0.9619294151	0.9170970948	0.8465887484	1.010048154
0.2	0.9757704592	0.8982109325	0.8552507831	0.995450198
0.3	0.9909982392	0.8786888364	0.8641295661	0.9804630895
0.4	1.0067380590	0.8585003366	0.8732341325	0.9650715764
0.5	1.0230170020	0.8376132379	0.8825740462	0.9492596408

The effect of suction parameter (c) on the rate of heat transfer between the channel walls and the working fluid is displayed in table II. It is observed that; the Nusselt numbers on the wall stationed at y = 0 increases with increase in c where as it is seen to decrease with increase in c on the plate kept at y = 1. Moreover, the Nusselt numbers on the wall positioned at y = 0 is seen to increase with increase in R while it is viewed to decrease with increase in R on the plate kept at y = 1.

CONCLUSION:

The study investigated natural convection flow with thermal radiation through a vertical porous channel filled with porous materials. The differential equations governing the fluid flow were transformed from partial differential equations into ordinary differential equations using similarity transformation. The resulting differential equations were solved semi-analytically using ADM. The semi-analytical solutions were simulated using computer algebra software "MAPLE'' and the results were presented on graphs and discussed. Effects of varying physical parameters on the skin friction and Nusselt number between the working fluid and the channel walls were presented on table I – II and discussed. Our result shows that, the fluid velocity increases with increase in Da and thermal radiation while the fluid temperature decreases with increase in suction parameter (c) where as it was observed to increase with increase in thermal radiation.

Nomenclature

q_r	Radiative heat flux
G_r	Grashof number
Pr	Prandtl number
Da	Darcy Number
Γ	Viscosity ratio
g	Gravitational Acceleration
t	Dimensionless Time
U_0	Wall velocity
T_w	Wall Temperature
T_0	Initial Temperature of the fluid
R	Radiation Parameter
h	Channel width
u	Dimensional velocity
y	Dimensionless horizontal coordinate
$T^{'}$	Dimensional fluid temperature

T Dimensionless fluid temperature

c Suction Parameter

k Thermal conductivity of fluid

K Porosity of the porous media

u, v Dimensionless velocity of the fluid

Nu_0 Nusselt number on the plate at $y = 0$

Nu_1 usselt number on the plate at $y = 1$

τ_0 Skin friction on the plate at $y = 0$

τ_1 Skin friction on the plate at $y = 1$

Greek symbols

α Thermal diffusivity

σ Stefan – Boltzmann constant

β Volumetric expansion coefficient

γ Kinematic viscosity of the fluid

γ_{eff} Kinematic viscosity due to porous medium

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