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METRICS AND METRIC SPACES OF SOFT MULTISETS

Isah, A. I.

Department of Mathematical Sciences, Kaduna State University, Kaduna-Nigeria

*Corresponding authors' email: aisah204@gmail.com

ABSTRACT

The theory of Soft set found applications in so many fields including multiset theory to obtain soft multisets. These theories together with some of their properties were presented. Moreover, considering the various applications of metric spaces in various fields; Metrics and metric spaces of soft multisets with some of their attributes were introduced. However, it was discovered that only pseudo-metric spaces could favorably be formulated. Moreover, soft multiset ordering was also presented.

Keywords: Metric, Multiset, Ordering, Soft multiset, Soft set

INTRODUCTION

Multiset which is a formulation of the general idea of a set was applied in many disciplines including fuzzy sets, soft sets etc., (Tokat et al. 2015; Ifetoye et al. 2022). The theory of multiset and its applications can be found in (Girish and Sunil 2009; Hickman, 1980; Monro, 1987, Blizard, 1989, Singh and Isah, 2016).

Alkhazaleh (2011) first introduced the theory of soft multisets (soft msets, for short).Thereafter, other scholars such as (Babitha and Sunil, 2013; Majumdar, 2012; Neog and Kut, 2012; Tokat and Osmanoglu, 2013; Isah, 2019; Osmanoglo and Tokat, 2014; Tokat et al., 2015) contributes their quota to the development of the theory in their own perspective which immensely boost the field. Moreover, considering the various applications of metric spaces in various fields including, internet search engines, image classification, signal analysis and processing; Metrics and metric spaces of soft multisets were introduced in this paper. In addition, soft multiset ordering was also presented.

Preliminaries

Multisets (Girish and Sunil, 2009)

Definition 1 Let X be a set, then a multiset M drawn from X is expressed as M or C_M defined as $C_M: X \longrightarrow \mathbb{N}$, where $a \in \mathbb{N} \Longrightarrow a \geq 0$. Note that for $x \in X$, $C_M(x)$ represent the number of occurrences of x in M called its characteristic value. If $\forall x \in X$, $C_M(x) = 0$ or 1, then M is a set. Moreover, if $\forall x \in X$, $C_M(x) = 0$, we say M is empty.
The cardinality of M written $|M|$ or $Card\ M = \sum_{x \in X} C_M(x)$.

Definition 2 Let X be a set and M an mset over X. The root or support set of M written as M^* is defined as $M^* = \{x \in X : CM(x) > 0\}$.

Definition 3 Let M_1 and M_2 be msets over a set X, then M_1 is a sub-mset of M_2, denoted $M_1 \subseteq M_2$ if $C_{M_1}(x) \leq C_{M_2}(x)$, $\forall x \in X$. M_1 is a proper sub-mset of M_2 written $M_1 \subset M_2$ if $C_{M_1}(x) \leq C_{M_2}(x)$, $\forall x \in X$ and $\exists$ at least one $x \in X$ such that $C_{M_1}(x) < C_{M_2}(x)$. M_1 and M_2 are said to be equal written $M_1 = M_2$ if $M_1 \subseteq M_2$ and $M_2 \subseteq M_1$.

Definition 4 Let M_1 and M_2 be msets over a set X, then their union, denoted by $M_1 \cup M_2 = \max\{C_{M_1}(x), C_{M_2}(x)\}$ and their intersection, denoted by $M_1 \cap M_2 = \min\{C_{M_1}(x), C_{M_2}(x)\}$.

Definition 5 Let M_1 and M_2 be msets over a set X. Then their Cartesian product is defined as $M_1 \times M_2 = \{(m/x, n/y)/mn : x \in^m M_1, y \in^n M_2\}$.

Definition 6 A sub-mset R of $M \times M$ is called an mset relation on M if every member $(m/x, n/y)$ of R has a count the product of $C_1(x, y)$ and $C_2(x, y)$. m/x related to n/y is denoted by $m/xRn/y$.

Definition 7 An mset relation R on a multiset M is said to be reflexive if $\forall\ m/x \in M$ we have $m/xRm/x$, antisymmetric if $m/xRn/y$ and $n/yRm/x$ implies $m/x = n/y$, for all $m/x, n/y \in M$. It becomes transitive if $m/xRn/y, n/yRk/z$ implies $m/xRk/z$ for all $m/x, n/y, k/z \in M$.

Definition 8 Let R be an mset relation on a multiset M. R is said to be a quasimset order also known as pre-mset order if it is both reflexive and transitive. It is an mset order otherwise known as a partially ordered mset relation if it is also antisymmetric. The pair (M, R) is said to be an ordered mset or partially ordered multiset (pomset).

Definition 9 The mset relation R is referred to as total mset order otherwise known as a linear mset order on M if R is an mset order and for every $m/x \neq n/y \in M$ either $m/xRn/y$ or $n/yRm/x$.

Soft Set (Molodtsov, 1999; Maji, et al., 2003; Ali et al., 2009; Sezgin and Atagun, 2011)

Definition 10 Let U be universe set and E a set of attributes pertaining U. If $P(U)$ is a power set of U and $A \subseteq E$, a pair (F, A) said to be a Soft set over U, with F is a mapping $F: A \longrightarrow P(U)$.
(F, A) is called a relative null soft set with regard to A, denoted by Φ_A, if $F(e) = \emptyset\ \forall e \in A$, it is a relative whole soft set pertaining A, denoted by U_A, if $F(e) = U\ \forall e \in A$. The relative whole soft set U_E pertaining E is called the absolute soft set over U.

Definition 11 Let (F, A) and (G, B) be soft sets over a common universe U, we say that (F, A) is a soft subset of (G, B), denoted $(F, A)\ \tilde{\subseteq}\ (G, B)$, if

(i) $A \subseteq B$, and
(ii) $\forall e \in A, F(e) \subseteq G(e)$.

Definition 12 Let (F, A) and (G, B) be soft sets over a common universe U, we say that

i. The union of (F, A) and (G, B), written $(F, A)\ \tilde{\cup}\ (G, B)$, is a soft set (H, C)
ii. where $C = A \cup B$ and $\forall e \in C$,

$$H(e) = \begin{cases} F(e), & e \in A - B \\ G(e), & e \in B - A \\ F(e) \cup G(e), & e \in A \cap B. \end{cases}$$

iii. The extended intersectionof (F, A) and (G, B), written $(F, A)\ \tilde{\cap}\ (G, B)$, is a soft set (H, C) where $C = A \cup B$ and $\forall e \in C$,

$$H(e) = \begin{cases} F(e), & e \in A - B \\ G(e), & e \in B - A \\ F(e) \cap G(e), & e \in A \cap B. \end{cases}$$

Definition 13 Let (F, A) and (G, B) be two soft sets over a common universe U such that $(F, A) \tilde{\subseteq} (G, B)$. Then, the relative difference of (F, A) and (G, B) is defined as $(G, B) - (F, A) = (H, E)$ where $H(e) = \{G(e) - F(e)\}, \forall e \in B$.

Definition 14 Let $E = \{e_1, e_2, \ldots, e_n\}$ be a set of parameters. The NOT set of E denoted by $\urcorner E$ is defined by $\urcorner E = \{\urcorner e_1, \urcorner e_2, \ldots, \urcorner e_n\}$ where $\urcorner e_i =$ not e_i, $\forall i \in \{1, 2, \ldots, n\}$.

Definition 15 The complement of' a soft set (F, A) written $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \urcorner A)$ where $F^c: \urcorner A \longrightarrow P(U)$ is a mapping given by $F^c(\alpha) = U - F(\urcorner\alpha)$ for all $\alpha \in \urcorner A$. F^c is said to be the soft complement function of F. It is clear $(F^c)^c = F$ and $((F, A)^c)^c = (F, A)$.

Definition 16 The relative complement of a soft set (F, A) written $(F, A)^r$ is defined by $(F, A)^r = (F^r, A)$ where $F^r: A \longrightarrow P(U)$ a mapping given by $F^r(\alpha) = U - F(\alpha)$ for all $\alpha \in A$. Clearly, $(F, A)^r = U_E -_R (F, A)$.

Definition 17 The restricted intersection of (F, A) and (G, B) written $(F, A) \cap_R (G, B)$, is a soft set (H, C) where $C = A \cap B$ and $\forall e \in C, H(e) = F(e) \cap G(e)$.

Definition 18 The restricted union of (F, A) and (G, B) written $(F, A) \cup_R (G, B)$, is a soft set (H, C) where $C = A \cap B$ and $\forall e \in C, H(e) = F(e) \cup G(e)$.

Soft Multiset (Tokat et al., 2015; Isah, 2018)

Definition 19 Let U be a universe multiset, E set of attributes and $A \subseteq E$. Then, a pair (F, A) or F_A is said to be a Soft multiset with F a mapping $F: A \longrightarrow P^*(U)$. $F(e)$ is expressed as $C_{F(e)}: U^* \longrightarrow \mathbb{N}$ for every $e \in A$,

For example, let $U = \{7/x, 9/y, 8/z\}$, $A = \{e_1, e_2\}$. If a soft multiset $(F, A) = \{(e_1, \{7/x, 3/y\}), (e_2, \{1/x, 2/z\})\}$, we have $C_{F(e_1)}(x) = 7, C_{F(e_1)}(y) = 3, C_{F(e_1)}(z) = 0, C_{F(e_2)}(x) = 1, C_{F(e_2)}(y) = 0, C_{F(e_2)}(z) = 2$.

Definition 20 Let U be a universe multiset, E a set of attributes and $A \subseteq E$.Then

i. (F, A) is called an empty or null soft mset, denoted $\Phi_\emptyset$ or $(F, A)_\emptyset$ if $A = \emptyset$.
ii. (F, A) is said to be a relative null soft mset pertaining A, denoted Φ_A or $(F, A)_{\emptyset A}$ if $F(e) = \emptyset, \forall e \in A$.
iii. (F, A) is said to be a relative semi-nullsoft multiset pertaining A, denoted Φ_{A1} or $(F, A)_{\emptyset 1}$ if $\exists e \in A$ such that $F(e) = \emptyset$.
iv. (F, A) is said to be a relative absolute soft multiset pertaining A, denoted U_A or $(F, A)_U$ if $F(e) = U, \forall e \in A$.
v. (F, E) is said to be a relative null soft multiset pertaining E, denoted Φ_E or $(F, E)_\emptyset$ if $F(e) = \emptyset, \forall e \in E$.
vi. (F, A) is said to be a relative semi-absolute soft multiset pertaining A, denoted U_{A1} or $(F, A)_{U1}$ if $\exists e \in A$ such that $F(e) = U$.
vii. (F, E) is said to be semi-absolute soft multiset denoted U_{E1} or $(F, E)_{U1}$ if $\exists e \in E$ such that $F(e) = U$.
viii. (F, E) is said to be the absolute soft multiset denoted U_E or $(F, E)_U$ if $F(e) = U, \forall e \in E$.

Definition 21 Let (F, A) and (G, B) be soft multisets over U. Then (F, A) is a soft submultiset of (G, B) presented as $(F, A) \sqsubseteq (G, B)$ if
(i) $A \subseteq B$
(ii) $C_{F(e)}(x) \leq C_{G(e)}(x), \forall x \in U^*, \forall e \in A$.
$(F, A) = (G, B) \Leftrightarrow (F, A) \sqsubseteq (G, B)$ and $(G, B) \sqsubseteq (F, A)$.
Moreover, if $(F, A) \sqsubseteq (G, B)$ and $(F, A) \neq (G, B)$then (F, A) is said to be a proper soft submultiset of (G, B) and (F, A) is a whole soft submultiset of (G, B) if $C_{F(e)}(x) = C_{G(e)}(x), \forall x \in U^*, \forall e \in A$.

Definiton 22 Let (F, A) and (G, B) be soft multisets over U. Then

i. The union of (F, A) and (G, B) is a soft set (H, C) where $C = A \cup B$ and $\forall e \in C, C_{H(e)}(x) = \max\{C_{F(e)}(x), C_{G(e)}(x)\}, \forall x \in U^*$.
ii. The intersection of (F, A) and (G, B) is a soft set (H, C) where $C = A \cap B$ and $\forall e \in C, C_{H(e)}(x) = \min\{C_{F(e)}(x), C_{G(e)}(x)\}, \forall x \in U^*$.
iii. The complement of (F, A), written $(F, A)^C$, is defined by $(F, A)^C = (F^C, A)$ where $F^C: A \longrightarrow P^*(U)$ a mapping given by $F^C(e) = U \backslash F(e), \forall e \in A$ with $C_{F^C(e)}(x) = C_U(x) - C_{F(e)}(x), \forall x \in U^*$.
iv. Their difference is defined as $(F, A) \backslash (G, B) = (H, E)$ with $C_{H(e)}(x) = \max\{C_{F(e)}(x) - C_{G(e)}(x), 0\}, \forall e \in E, \forall x \in U^*$.

Definition 23 Let (F, A) and (G, B) be soft multisets over U, such that $(F, A) \sqsubseteq (G, B)$, then the relative complement of (F, A) to (G, B) is a soft multiset $(G, B) - (F, A) = (H, C)$ where $C_{H(e)}(x) = \{C_{G(e)}(x) - C_{F(e)}(x)\}, \forall e \in B, \forall x \in U^*$.

Definition 24 Let (F, A) be a soft multiset. Then its power soft multiset written $P(F_A)$ or $P(F, A)$ is defined as the soft multiset of all soft submultisets of (F, A).

Definition 25 Let $(G, B) \in P(F, A)$, then the relative complement of (G, B) in $P(F, A)$ denoted $\overline{(G, B)}$ is a mapping $\overline{(G, B)}: B \longrightarrow P^*(U)$ defined by $C_{\overline{G(e)}}(x) = C_{F(e)}(x) - C_{G(e)}(x), \forall e \in A, \forall x \in U^*$.

Theorem 1 Let (F, A) be a soft multiset and $P(F, A)$ be its power soft multiset. Let $(G, B), (H, C) \in P(F, A)$ then
(i) $\overline{(G, B) \sqcup (H, C)} = \overline{(G, B)} \sqcap \overline{(H, C)}$
(ii) $\overline{(G, B) \sqcap (H, C)} = \overline{(G, B)} \sqcup \overline{(H, C)}$

Proof

(i) Let $(G,B) \sqcup (H,C) = (K, B \cup C)$, then $\overline{(G,B) \sqcup (H,C)} = \overline{(K, B \cup C)}$

Let $(e, K(e)) \in \overline{(G,B) \sqcup (H,C)}$

$$\Rightarrow (e, K(e)) \in \overline{(K, B \cup C)}$$
$$\Rightarrow (e, K(e)) \in (C_{F(e)}(x) - C_{K(e)}), \forall e \in A, \forall x \in U^*$$
$$\Rightarrow (e, K(e)) \in (C_{F(e)}(x) - (C_{G(e)} \cup C_{H(e)})), \forall e \in A, \forall x \in U^*$$
$$\Rightarrow (e, K(e)) \in ((C_{F(e)}(x) - C_{G(e)}) \cap (C_{F(e)}(x) - C_{H(e)})), \forall e \in A, \forall x \in U^*$$
$$\Rightarrow (e, K(e)) \in \overline{(G,B)} \sqcap \overline{(H,C)}$$

i.e., $\overline{(G,B) \sqcup (H,C)} \sqsubseteq \overline{(G,B)} \sqcap \overline{(H,C)}$

Conversely, let $\overline{(G,B)} \sqcap \overline{(H,C)} = \overline{(J, B \cap C)}$

Suppose $(e, J(e)) \in \overline{(G,B)} \sqcap \overline{(H,C)}$

$$\Rightarrow (e, J(e)) \in \overline{(J, B \cap C)}$$
$$\Rightarrow (e, J(e)) \in (C_{F(e)}(x) - C_{J(e)}), \forall e \in A, \forall x \in U^*$$
$$\Rightarrow (e, K(e)) \in (C_{F(e)}(x) - (C_{G(e)} \cap C_{H(e)})), \forall e \in A, \forall x \in U^*$$
$$\Rightarrow (e, J(e)) \in ((C_{F(e)}(x) - C_{G(e)}) \cup (C_{F(e)}(x) - C_{H(e)})), \forall e \in A, \forall x \in U^*$$
$$\Rightarrow (e, J(e)) \in \overline{(G,B) \sqcup (H,C)}$$

i.e., $\overline{(G,B)} \sqcap \overline{(H,C)} \sqsubseteq \overline{(G,B) \sqcup (H,C)}$

The proof of (ii) is analogous to (i).

Remark 1 For every $(G,B) \in P(F,A)$ we have $C_{G(e)}(x) \leq C_{F(e)}(x), \forall e \in B, \forall x \in U^*$. Then

(i) $\overline{\overline{(G,B)}} = (G,B)$

(ii) , $\overline{(G,B)} \sqcup \overline{(G,B)} = \overline{(G,B)}$ and $\overline{(G,B)} \sqcap \overline{(G,B)} = \overline{(G,B)}$

However, neither $\overline{(G,B)} \sqcap (G,B) = \Phi_B$ nor $\overline{(G,B)} \sqcup (G,B) = (G,B)$ hold, in general.

Metrics and Metric spaces of soft multiset

Definition 26 Let (Q,S) be a soft multiset over U with $S \subseteq E$ an attribute set, then (Q,S) is presented as

$(Q,S) = \{(s_1, Q(s_1)), (s_2, Q(s_2)), (s_3, Q(s_1)), \ldots\}$.

Let the cardinality of an element $(s_k, Q(s_k))$ of (Q,S) be denoted by $|(s_k, Q(s_k))|$

i.e., $|(s_k, Q(s_k))| = \sum_{s_k \in S} C_{Q(s_k)}(z), \forall z \in U^*$. The maximum cardinality of an element of (Q,S) is called its height denoted hgt(Q,S). The cardinality of (Q,S) denoted $|(Q,S)| = \sum_{s_i \in S} |(s_i, Q(s_i))| = \sum_{s_i \in S} C_{Q(s_i)}(z) \ \forall z \in U^*, \ \forall i$.

Definition 27 Let $\Omega = SM(U)$ denote the collection of all soft msets over a universe multiset U and an attribute set E. Then $Q_S \in \Omega$ is called regular or constant if $\forall s \in S, \forall\, y, z \in U^*, \ C_{Q(s)}(y) = C_{Q(s)}(z)$ and $C_{Q(s)}(y)$ is said to be its height. Moreover, the relative null soft multiset Φ_S is a regular soft mset with height 0.

Theorem 2 Let $F_A, R_B \in \Omega$ and the distance d be defined as $d(F_A, R_B) = |hgtF_A - hgtR_B|$. Then d is a pseudo or semi-metric on Ω and (Ω, d) is a pseudo or semi-metric space.

Proof

Let $F_A, R_B, T_C \in \Omega$, then

$d(F_A, R_B) = |hgtF_A - hgtR_B| \geq 0$.

$d(F_A, R_B) = |hgtF_A - hgtR_B| = |hgtR_B - hgtF_A| = d(R_B, F_A)$.

$d(F_A, T_C) = |hgtF_A - hgtT_C| = |hgtF_A - hgtR_B + hgtR_B - hgtT_C|$.

$$\leq |hgtF_A - hgtR_B| + |hgtR_B - hgtT_C|.$$
$$\leq d(F_A, G_B) + d(G_B, H_C).$$

Moreover,$d(F_A, F_A) = |hgtF_A - hgtF_A| = 0$.

Thus d is a pseudo or semi-metric on Ω and (Ω, d) is a pseudo or semi-metric space. Moreover, $d(F_A, R_B) = 0$ iff $F_A = R_B$ fails. Nevertheless, if $F_A = R_B \Rightarrow d(F_A, R_B) = 0$.

Theorem 3 Let $J_A, L_B \in \Omega$ and the distance d be defined by $d(J_A, L_B) = max|C_{J(e)}(z) - C_{L(e)}(z)| \forall e \in E, \forall z \in U^*$. Then d is a semi-metric on Ω and (Ω, d) is a semi-metric space.

Proof

Let $J_A, L_B, M_C \in \Omega$, then $\forall s \in E, \forall z \in U^*$

$d(J_A, L_B) = max|C_{J(s)}(z) - C_{L(s)}(z)| \geq 0$.

$d(J_A, L_B) = max|C_{J(s)}(z) - C_{L(s)}(z)| = max|C_{L(s)}(z) - C_{J(s)}(z)| = d(G_B, F_A)$.

$$d(J_A, M_C) = max|C_{J(s)}(z) - C_{M(s)}(z)|$$
$$= max|C_{J(s)}(z) - C_{L(s)}(z) + C_{L(s)}(z) - C_{M(s)}(z)|$$
$$\leq max|C_{J(s)}(z) - C_{L(s)}(z)| + \max|C_{L(s)}(z) - C_{M(s)}(z)|$$
$$\leq d(J_A, L_B) + d(L_B, M_C).$$

Observe that$d(J_A, J_A) = max|C_{J(s)}(z) - C_{J(s)}(z)| = 0$.

Thus d is a semi-metric on Ω and (Ω, d) is a semi-metric space. Moreover, if $J_A = L_B$ we have $d(J_A, L_B) = 0$, however $d(J_A, L_B) = 0$ does not always imply $J_A = L_B$.

Remark 2 We get a similar result if $C_{J(e)}(z)$ is replaced with $|J_A|$ and $C_{L(e)}(z)$ with $|L_B|$.

Theorem 4 Let $P_A, Q_B \in \Omega$ and the distance d be defined as $d(P_A, Q_B) = \sum |C_{P(s)}(z) - C_{Q(s)}(z)|\ \forall s \in E, \forall z \in U^*$. Then d is a semi-metric on Ω and (Ω, d) is a semi-metric space.

Proof
Let $P_A, Q_B, R_C \in \Omega$, then $\forall s \in E, \forall z \in U^*$

$$d(P_A, Q_B) = \sum |C_{P(s)}(z) - C_{Q(s)}(z)| \geq 0$$

$d(P_A, Q_B) = \sum |C_{P(s)}(z) - C_{Q(s)}(z)| = \sum |C_{Q(s)}(z) - C_{P(s)}(z)| = d(Q_B, P_A)$.

$$\begin{aligned}(P_A, R_C) = \sum |C_{P(s)}(z) - C_{R(s)}(z)| &= \sum |C_{P(s)}(z) - C_{Q(s)}(z) + C_{Q(s)}(z) - C_{R(s)}(z)| \\ &\leq \sum |C_{P(s)}(z) - C_{Q(s)}(z)| + \sum |C_{Q(s)}(z) - C_{R(s)}(z)| \\ &\leq d(P_A, Q_B) + d(Q_B, R_C).\end{aligned}$$

$d(P_A, P_A) = \sum |C_{P(s)}(z) - C_{P(s)}(z)| = 0$.

Thus d is a semi-metric on Ω and (Ω, d) is a semi-metric space. Moreover, if $P_A = Q_B$ we have $d(P_A, Q_B) = 0$, however $d(P_A, Q_B) = 0$ does not always imply $P_A = Q_B$.

Definition 28 Diameter of a soft multiset
Let (Ω, d) be a semi metric space and let $Q_S \in \Omega$, then the diameter of Q_S, denoted $\delta(Q_S)$, can be defined by $\delta(Q_S) = sup\{d((s_i, Q(s_i)), (s_j, Q(s_j))) | (s_i, Q(s_i)), (s_j, Q(s_j)) \in Q_S, \forall i, j\}$, where $d\left((s_i, Q(s_i)), (s_j, Q(s_j))\right) = \left||(s_i, Q(s_i))| - \left|(s_j, Q(s_j))\right|\right|$.

Definition 29 Distance between two soft multisets
Let (Ω, d) be a semi metric space and $F_A, R_B \in \Omega$, then the distance between F_A and R_B, denoted $\rho(F_A, R_B)$, can be defined by $\rho(F_A, R_B) = inf\ \{d((s_r, F(s_r)), (s_q, R(s_q))) | (s_r, F(s_r)) \in F_A,\ (s_q, R(s_q)) \in R_B, \forall r, q\}$ where $d((s_r, F(s_r)), (s_q, R(s_q))) = \left||(s_r, F(s_r))| - \left|(s_q, R(s_q))\right|\right|$.

Remark 3 If $F_A \cap R_B \neq \emptyset$ we have $\rho(F_A, R_B) = 0$, however, the converse may not hold. $|F_A|$ can be regarded as the distance $\rho(F_A, \emptyset_A)$ of F_A from the relative null soft multiset.

Soft multiset ordering
Definition 30 Let $SM(U)$ be the collection of all soft msets over a totally ordered multiset U and $F_A, G_B \in SM(U)$. Let $(e_i, F(e_i))$ be an element of F_A and $(e_j, G(e_j))$ an element of G_B. Suppose$\chi(F_A)$ and $\chi(G_B)$ respectively, denotes a sequence of the elements of F_A and G_B in descending order of their cardinalities i.e.,$|e_1, F(e_1)| \geq |e_2, F(e_2)| \geq \cdots \geq |e_n, F(e_n)|$ and $|e_1, G(e_1)| \geq |e_2, G(e_2)| \geq \cdots \geq |e_n, G(e_n)|$. Then an ordering $\ll$ on $SM(U)$ can be considered as
F_A Dominates G_B presented as $F_A \ll G_B$ if $\chi(F_A) \overset{lex}{>} \chi(G_B)$.
Meaning if $\chi(F_A)$ is lexicographically greater than $\chi(G_B)$.

For example, let $F_A = \{(a_1, \{1/r, 2/s, 1/w\}), (a_2, \{1/r, 1/w\}), (a_3, \{1/r, 4/y\}), (a_4, \{3/r, 4/y\})\}$ and $G_B = \{(b_1, \{5/y\}), (b_2, \{5/r, 2/s\}), (b_3, \{2/w, 1/r\}), (b_4, \{2/w\}), (b_5, \{1/r, 1/y, 1/w\})\}$ be soft multisets over $U = \{7/r, 9/y, 8/s, 4/w\}$ we have
$\chi(F_A) = \{ (a_4, \{3/r, 4/y\}), (a_3, \{1/r, 4/y\}), (a_1, \{1/r, 2/z, 1/w\}), (a_2, \{1/r, 1/w\})\}$ and
$\chi(G_B) = \{(b_2, \{5/r, 2/s\}), (b_1, \{5/y\}), b_3, \{2/w, 1/r\}), (b_5, \{1/r, 1/y, 1/w\}), (b_4, \{2/w\})\}$. Then $F_A \ll F_B$ as $\chi(F_A) \overset{lex}{>} \chi(G_B)$.

CONCLUSION
After presenting multisets, soft sets and soft multisets with some of their properties, soft multiset ordering and pseudo metrics of soft multiset were formulated. However, it is recommended that in future full metric spaces of soft multisets could be investigated.

REFERENCES
Ali,M. I., Feng, F., Liu, X., Min, W. K. and Shabir, M. (2009). On some new operations in soft set theory, Computers and Mathematics with Applications 57, 1547–1553.

Alkhazaleh, S, Salleh, A. R. and Hassan, N. H. (2011). Soft multisets theory, Applied Mathematical Sciences 72 (5) (2011) 3561–3573.

Babitha, K. V. and John,S. J. (2013). On soft multi sets, Annals of Fuzzy Mathematics and Informatics 5 (1) 35–44

Blizard, W. D. (1989). Multiset Theory, Notre Dame Journal of Formal Logic 30 (1) 36 – 66.

Girish and Sunil J. J. (2009). General relations between partially ordered multisets and their chains and antichains, Mathematical Communications 14 (2) 193 – 205.

Hickman, J. L., A note on the concept of multiset, Bulletin of the Australian Mathematical Society, 22 (1980) 211-217.

Ifetoye, O. O., Isah, A. I. and Tella, Y. (2022). Some Properties of Fuzzy Multigroups, FUDMA Journal of Sciences, 6 (1) (2022) 278 – 281.

Isah, A. I. (2019). Soft multiset in a new perspective, Annals of Fuzzy Mathematics and Informatics, 18 (3) 309-315.

Isah, A. I. (2018). Some Properties of Soft Multiset, Science World Journal 13 (4) 44-46.

Maji, P. K., Biswas, R. and Roy, A. R. (2003). Soft Set Theory, Computers and Mathematics with Applications (45) 555-562.

Majumdar, P. (2012). Soft multisets, Journal of Mathematics and Computer Science 2 (6) 1700–1711.

Molodtsov, D. (1999). Soft set theory-First results, Computers and Mathematics with Applications, 37, 19-31.

Monro, G.P. (1987). The concept of multiset. Zeitschriftfür mathematische Logik und Grundlagen der mathematic [Mathematical Logic Quarterly] 33(2), 171–178.

Neog, T. J. and Sut, D. K. (2012). On soft multisets theory, International Journal of Advanced Computer and Mathematical Sciences 3 (3)295–304

Osmanoglu, I. and Tokat, D. (2014). Compact soft multi spaces, European Journal Of Pure And Applied Mathematics 7 (1) 97–108

Singh, D. and Isah, A. I. (2016). Mathematics of Multisets: a unified Approach, Afrika Matematika, 27, 1139-1146.

Tokat, D. and Osmanoglu, I. (2013). Connectedness on soft multi topological spaces, Journal of New Results in Science (2) 8–18.

Tokat, D., Osmanoglu, I. and Ucok, Y. (2015). On soft multi semi-continuous functions, Annals of Fuzzy Mathematics and Informatics 10 (5) 691–704.

Sezgin, A. and Atagun, A. O. (2011). On operations of soft sets, Computers and mathematics with applications, 60, 1840-1849.