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REINFORCED CONCRETE BEAM AT ULTIMATE LIMIT STATE CONSIDERING VARIABILITY IN THE CONCRETE MIX DESIGN PARAMETERS

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ABSTRACT

Variability in design parameters of Civil Engineering infrastructure is inevitable at implementation in most practical situations, the effect of which can best be determined at design stage using reliability concepts. This study used First Order Reliability Method (FORM) to determine the effect of variation in Nano Engineered Concrete (NEC) mix design parameters on structural safety of Nano Engineered Reinforced Concrete (NERC) beams at ultimate limit state using reliability based sensitivity analysis. The variability in NEC mix design parameters was incorporated into EN1992-1-1(2008) design formulations using predictive models of NEC characteristic compressive strength developed on the basis of experimental data with the aid of DataFit statistical package. FORTRAN based subroutines of the NERC beam performance functions were developed and used for the reliability sensitivity analysis at ultimate limit state. Results indicate that variability in NEC mix design parameters affect structural safety of NERC beams in accordance with how the mix design parameter contributes to characteristic compressive strength development. Moreover, increase in nanosilica dosage beyond optimal value was found to have negative effect on structural safety of NERC beams. The study found that compressive strength contributes to tensile strength of NERC beams through composite action. Furthermore, the study suggested that the effect of variability in concrete mix design parameters be incorporated in design formulations of Civil Engineering Codes of Practice to aid estimation of safety of structural elements to be designed and produced with NEC for improved safety, sustainability and resilience of Civil Engineering infrastructure.

Keywords: variability, nano engineered concrete, nano engineered reinforced concrete beam, safety, ultimate limit state, mix design parameters

INTRODUCTION

Concrete is the most widely used material on the surface of the earth after water, whose usage is increasing daily with increase in the earth population and infrastructural development to support increasing economic activities and the need to repair ageing infrastructure (Ashani *et al.*, 2015). Concrete is a highly assorted material prepared by the combination of finely powdered cement, aggregates of various sizes and water with inherent physical, chemical and mechanical properties (Torgal et *al.*, 2013). The design of concrete structures consumes almost the total cement production in the world, but when concrete structures are exposed to severe environments; their performance become inferior, thereby leading to damage (Rajkumar *et al.*, 2016). However, a better understanding of precise engineering of an extremely complex structure of cement based materials at nano-level will apparently result in a new generation of concrete that is stronger and more durable, with desired stress strain behaviour and possibly possessing a range of newly introduced properties such as increased stress bearing ability, reduced permeability, as well as reduced moisture absorption (Rajkumar *et al.*, 2016).

Nanotechnology is the science that deals with particles which are less than 100nm. It is widely regarded as one of the twentieth century's key technologies whose economic weightage is speedily on the rise (Ashani *et al.*, 2015). Novel properties of materials manufactured on the nano – scale, like nanosilica, can be utilized for the benefit of construction infrastructure, as the application of nano materials in concrete can potentially change the service life and life - cycle cost of construction infrastructure (Rajkumar *et al.*, 2016). According to Sobolev and Sanchez (2010) and Sobolev *et al.* (2016); concrete containing nanoparticles such as Nanosilica (nS) are called Nano Engineered Concrete (NEC), which have the potential of providing increased load bearing ability, reduced brittleness, increased toughness, reduced permeability and high durability. Therefore, this work considered NEC due to its perceived prospect in providing classical strength, improved durability, and its potential in leading to sustainability in concrete production and usage.

Beams are structural members with the primary function of resisting bending (AISC-360, 2005). As defined by Nowak and Collins (2000) in Adamu (2014); a limit state is a boundary between desired and undesired performance of a structure, thus, Ultimate Limit State (ULS) has to do with a condition or state that can cause collapse of a structure or its parts. Abejide and Adamu (2013) opined that, Ultimate Limit State Design (ULSD) in beams considers flexure as the main failure criteria to which beams should be sized in order to satisfy both flexural and shear demands, but the authors showed that shear could also be a critical failure mode on whose basis beams could be sized at ULS in order to satisfy the requirements of achieving safe and economical designs. Thus, this study considered a singly reinforced concrete beam in carrying out reliability based sensitivity analysis at ULS being a major structural element in structural systems. The choice of a singly reinforced concrete beam was to ensure minimal reinforcement that could allow better assessment of the contribution of NEC in resisting applied stresses at ULS.

At ULS, failure modes such as shear and bending are considered in Reinforced Concrete (RC) design (Abejide, 2014). RC is a composite material formed by embedding reinforcing steel in concrete at desired location in line with the requirements of resisting applied bending and shear stresses. The arrangement in RC, as a composite material, was meant to utilize the natural ability of concrete in resisting compressive stresses within its carrying capacity as an artificial rock, and to cater for its weakness in resisting excessive applied bending and shear stresses. This means that, RC depend on the contribution of concrete in compression and shear, and reinforcing steel in tension and shear to support applied stresses that are above the carrying capacity of mass concrete sections. Therefore, it is essential to assess the safety level of designed RC sections like beam, so as to determine the contribution of concrete and how variability in mix design parameters affect the safety of RC structural elements produced with NEC. Moreover, since NEC is a new material required for sustainable Civil Engineering construction where reduced atmospheric carbon foot print is required (Venkat Rao *et al.*, 2014; Aly *et al.*, 2018; Behzadian and Shahrajabian, 2019), an assessment of the safety of RC elements to be produced with NEC is going to be vital to designers, sponsors and end users who are determined to ensure that scarce resources dedicated for infrastructural development projects are not wasted. Furthermore, a better understanding of how variability in NEC mix design parameters effect structural safety could encourage adoption and utilization of NEC in Civil Engineering designs and constructions. These considerations are part of what motivated this study, as there are presently no studies undertaken and reported in this aspect to best knowledge of the authors.

The United Kingdom Ministry of Defense (1991) defined reliability as the ability of an item to perform, or be capable of performing a required function without failure under stated conditions for a stated period on a unit of operation. According to Nowak (2004); structural reliability and probabilistic methods have continued to develop growing importance in modern Civil Engineering curricula in United States of America (USA) and across the world. They are currently used in the development of new generation design codes, evaluation of existing or proposed Civil Engineering structures and probability risk assessment. Stewart (1996) opined that a structure is only as strong as its 'weakest element', as a result, members with good measure of reliability do contribute significantly to the overall structural reliability.

Presence of uncertainties in the analysis and design of engineering structures has always been recognized. However, traditional approach as in Eurocodes simplified the problem, by considering the uncertain parameters to be deterministic and accounted for the uncertainties through the use of empirical safety factors derived based on past experience, but do not absolutely guarantee safety or satisfactory performance, neither do they provide information on the influence of different parameters in design (Halder and Mahadevan, 2000). Therefore, it is in the interest of the concrete industry and the engineering community to produce concrete structures that not only have an adequate safety margin against collapse, but also provide acceptable performance at minimum cost (Lee *et al.*, 2007). According to Qianru and Ann (2013), ensuring consistent safety levels requires a reliability-based approach that reconciles uncertainties.

Nigeria as a former British colony and as member of commonwealth nations has over time adopted British Codes of Practice such as BS 8110 and the likes for reinforced concrete design. Recent development has shown that all British Standard Codes of Practice have been withdrawn and replaced with Eurocodes released by European Committee for Standardization. According to Boussida *et al.* (2010); Eurocodes are a set of European Standards which provide common rules for the design of construction works; to check their strength and stability against live and extreme loads such as earthquakes and fire. As opined by Narayanan (2008); the European Union Eurocodes reflect the results of research in material technology and structural behavior in the last fifty years, Eurocodes incorporate all modern trends in structural design. They are among the most advanced suites of structural and material codes in the world, as they embody the collective experience and knowledge of the whole of Europe (Narayanan, 2008). Eurocodes are presently the main reference documents for product standards and European technical approvals for CE-markings of construction products in European market. With these developments, it is apparent that in the near future, Eurocode 2 (EC2) and its reference standards would replace BS8110 and accompanying standards, as the most used Reinforced Concrete Design (RCD) code in Nigeria and commonwealth nations; whose engineering practice is mostly guided by the provisions of British Standards Codes of Practice. Hence, the reliability based sensitivity analysis of a nano-engineered reinforced concrete beam considering variability in the concrete mix design parameters was done at ULS according to Eurocodes.

Concept of Reliability Analysis

To be able to have a better understanding of the concept of reliability analysis and failure probability, one could consider an example of demand ($\mathbf{X_2}$) and supply ($\mathbf{X_1}$) given by Ang and Tang (1984) and reproduced in Adamu (2014). Failure is said to have occurred in this example when demand is greater than or equal to supply, that is when: $\boldsymbol{X_1 - X_2 \leq 0}$. When carrying out reliability analysis, the variables $\mathbf{X_1}$ and $\mathbf{X_2}$ are normally considered to be random with probability density functions (Ang and Tang, 1984; Adamu, 2014). According to Ang and Tang (1984), as reported in Adamu (2014), the expression for the failure probability is as in Eq. (1) if the demand and supply are statistically independent.

$$\mathrm{P_f} = \int_{-\infty}^{+\infty}\left[1 - \mathrm{F}_{\mathrm{X_2}}(\mathrm{X_1})\right]\mathrm{F}_{\mathrm{X_1}}(\mathrm{X_1}) \qquad (1)$$

where $\mathrm{F}_{\mathrm{X_1}}(\mathrm{X_1})$ is the probability density of $\mathrm{X_1}$ at $\mathrm{x_1}$ *and* $\mathrm{F}_{\mathrm{X_2}}(\mathrm{X_1})$ is the cumulative distribution of $\mathrm{X_2}$ at $\mathrm{x_1}$.

When the supply and demand problem is formulated in terms of safety margin, it will result to Eq. (2) as provided in Ang and Tang, 1984 and Adamu, 2014.

$$\mathrm{G(X)} = X_1 - X_2 \qquad (2)$$

where $\mathrm{X_2}$ and $\mathrm{X_1}$ are normally distributed independent random variables.

Failure is said to have occurred if the condition in Eq. (3) is satisfied.

$$\mathrm{G(X)} = X_1 - X_2 \leq 0 \qquad (3)$$

G (X) can be considered as normally distributed random variable with probability density function f_G whose mean value can be obtained from Eq. (4), being that the safety margin G (X) is a function of two independent normally distributed random variables.

$$\mu_G = \mu_{X_1} - \mu_{X_2} \qquad (4)$$

The standard deviation of G(X) can be given by Eq. (5) obtained from Ang and Tang, 1984.

$$\sigma_G = \sqrt{\sigma_{X_1}^2 - \sigma_{X_2}^2} \qquad (5)$$

The failure probability can be given by Eq. (6) presented in Ang and Tang (1984) and Adamu (2014).

$$\mathrm{P_f} = \int_{-\infty}^{0} f_{G}(g)\, d_g = \mathrm{F_G}(\beta) \qquad (6)$$

If β is the number of standard deviations σ_G from the mean value μ_G to the failure region, then failure occurs when Eqs. (7) and (8) are satisfied.

$$\mu_G - \beta\sigma_G = 0 \qquad (7)$$

$$\beta = \frac{\mu_G}{\sigma_G} = \frac{\mu_{X_1} - \mu_{X_2}}{\sqrt{\sigma_{X_1} + \sigma_{X_2}}} \qquad (8)$$

The failure probability is given by Eq. (9) obtained after substitution and simplification (Ang and Tang, 1984; Adamu, 2014)

$$\mathrm{P_f} = F_G\left(\frac{\mu_G}{\sigma_G}\right) = 1 - \emptyset(\beta) \qquad (9)$$

where $\emptyset(\beta)$ is the standard normal cummulative density function evaluated at β**.** The quantity β is referred to as the reliability or safety index.

MATERIALS AND METHODS

Materials

This study used nanosilica (nS) with commercial name VK-SP15, whose PH, SiO_2 content, average particle size, and specific surface area were determined to be 7.60, 97.92%, 19.5nm and $265 m^2/g$ respectively. The work used crushed aggregate of 25mm maximum size and fine aggregate finer than 4.5mm sieve obtained from local suppliers in the production of NEC mixes. The absorption capacity, dry rodded unit weight, finess modulus, moisture content (SSD), specific gravity (SSD) of the fine aggregate were 2.0%, 1550kg/ m^3 , 2.6, 3.0%, 2.67 respectively. The coarse aggregate has absorption capacity, specific gravity (SSD), moisture content (SSD), impact value, crushing value, elongation and flackiness indices of 0.9%, 2.5, 0.89%, 17.4%, 27.9%, 30.20% and 20.4% respectively. The work used Portland Limestone Cement type II B-L produced in line with Nigeria Industrial Standard (NIS444-1:2003) by Cement Company of Northern Nigeria having standard consistency, soundness, initial setting time, final setting time and specific gravity of 30%, 2.0mm, 135minutes, 345minutes and 3.14 respectively. Hydroplast 500 superplastisizer produced by Armosil Limited was used to give the NEC mixes required workability. The aggregate grading conformed to the provisions of BS EN933-1:2012, BS EN12620:2013 and ASTM C33/C33M. The water used in production of NEC mixes and curing of specimens conformed to the requirements of EN1008:2002. In the reliability based sensitivity analysis, secondary materials were used such as: European Codes of Practice and their reference standards, the experimental data for characteristic cube compressive strength of NEC, the characteristic compressive strength predictive models developed from the experimental data, stochastic models, parameters of stochastic models, textbooks, research journals, conference proceedings, technical reports, dissertations and thesis.

Design and Validation of Concrete Mixes

The work used MS Excel (2007) to carry out concrete mix design calculations for two grades of concrete (grades 30 and 40) according to American Concrete Institute (ACI) guidelines for designing concrete mixes provide in ACI 211.2-98 (2004) and ACI 211.4R-93 (1998). The ACI concrete design guides were used for the mix design because of being the most recent and simpler to implement when compared with the British method provided in Building Research Establishment guides (BRE, 1988). The mix design targets mean cylindrical compressive strengths of 31.87 N/ mm^2 and 39.9N/ mm^2 derived from the target cube compressive strengths of 39.4N/ mm^2 and 49.8N/ mm^2 at 28days for grades 30 and 40 respectively. The design targets a slump value of 80mm using 25mm maximum aggregate sizes for the two concrete grades. The results of the trial concrete mix design are provided in Table 1.

Table 1: Trial Mix Proportions for Grade 30 and 40 Concrete Mixes

Grade of Concrete	W/C	Cement	Aggregates		Water	Superplastisizer	
			Coarse	Fine		%	Weight
		kg/m^3	kg/m^3	kg/m^3	kg/m^3		kg/m^3
30	0.50	390	963	835	163	2.0	7.80
40	0.35	557	963	688	170	2.0	11.14

The trial mixes were used to produce concrete samples according BS EN206:2013 and BS EN12350-2:2009 respectively for compressive strength and slump tests. Compressive strength test was carried out on the concrete samples at 28days of curing according to BS EN12390-3:2009. Adjustments were made to the trial concrete mixes for the two concrete grades according to Neville and Brooks (2010) until the desired characteristic compressive strengths and slump values were obtained. The characteristic cube strength was determined using Eq. (10) using measures of dispersion obtained from statistical analysis of experimental data with the aid of MS Excel (2007). The validated mix proportions for the two concrete grades are presented in Table 2. The validated mixes having various proportions of nanosilica for the concrete grades are presented in Table 3.

$$\mathrm{f_{cK}} = \mathrm{f_m} - 1.64\mathrm{S}_d \qquad (10)$$

where $\mathrm{f_{cK}}$, $\mathrm{f_m}$ and S_d are characteristic cube strength, mean strength and standard deviation respectively.

Table 2: Validated Mix Proportions for Grade 30 and 40 Concrete

Grade of Concrete	W/C	Cement	Aggregates		Water	Superplastisizer	
			Coarse	**Fine**		**%**	**Weight**
		kg/m^3	**kg/m^3**	**kg/m^3**	**kg/m^3**		**kg/m^3**
30	0.45	433	963	797	165	2.00	8.66
40	0.30	650	963	606	174	2.00	13.00

Table 3: Validated mixes containing varying nanosilica proportions

Grade	Des.	W/C	Cement	Nanosilica		Aggregate		Water	Superplastisizer	
			kg/m^3	%	kg/m^3	kg/m^3		kg/m^3	%	kg/m^3
						Fine	Coarse			
30	NSD0	0.45	433	0.0	0.0	797	963	165	2.00	8.70
	NSD0.5		431	0.5	2.2					
	NSD1		429	1.0	4.3					
	NSD1.5		427	1.5	6.5					
	NSD2		425	2.0	8.7					
	NSD2.5		423	2.5	10.8					
	NSD3		420	3.0	13.0					
40	NSD0	0.30	650	0.0	0.0	606	963	174	2.00	13.00
	NSD0.5		647	0.5	3.3					
	NSD1		644	1.0	6.5					
	NSD1.5		640	1.5	9.8					
	NSD2		637	2.0	13.0					
	NSD2.5		634	2.5	16.3					
	NSD3		631	3.0	19.5					

Sample Preparation for Compressive Strength Test

To determine the variation in compressive strength of the validated concrete mixes for the two concrete grades (Grades 30 and 40) without nanosilica and with varying nanosilica contents, concrete cubes having 100mm x 100mm x 100m size were produced according to BS EN12390-1:2012. Nanosilica dosages were kept at a range of 0 to 3% by weight of cementations materials as in Adamu *et al.* (2020a) and Adamu *et al.* (2020b) for each of the two concrete grades. In producing the mixes, the percentage of nanosilica in each mix was measured and placed in the mixing compartment of Altrad Mini Mix 130 concrete mixer, the required amount of water and superplastisizer were added and mixed thoroughly for five (5) minutes to facilitate proper dispersal of the nonosilica. This was necessary due to the high surface area of nS and the difficulty associated with its dispersal. The mixing progressed by adding the required amount of cement to the mixer and mixed at medium speed. Thereafter, the fine and coarse aggregates were added and mixed thoroughly to form a homogenous mix. 100mm x 100mm x 100mm oiled molds were used in casting the concrete cubes. After casting, the molds containing the concrete mixes were vibrated on a vibrating table. Demolding of the test samples was done after 24 hours of casting. The specimens were then placed in curing tanks and cured for 28days under laboratory condition. The concrete samples were tested for slump in accordance with BS EN12350-2:2009 to determine their workability.

Compressive Strength Test

The compressive strength test was done on the concrete samples having varying percentages of nS (0 to 3%) for the two concrete grades. The test was done at 28days of curing in line with the requirements of BS EN12390-3:2009; which requires the load rate of the testing machine to be kept at 0.6 ± 0.2MPa/Sec. Load was sustained on the cubes using Avery Denison Universal Testing Machine until the specimens failed. The recorded failure loads for the concrete samples, for each concrete grade and nS dosage were used to compute the compressive strength of each sample by dividing the failure load with the cubes cross-sectional area. To obtain the mean strength, an average of the compressive strength of three concrete cube samples produced for each concrete grade and nS dosage was taken according to BS EN12390-3:2009. The characteristic cube strengths for each concrete grade and nS dosage were calculated using Eq. (10) utilizing the computed measures of dispersion. The characteristic cube compressive strengths were converted to cylindrical compressive strengths using Eq. (11) from Domone (2010) to allow for design according to Eurocodes. The experimental work was done at Concrete Laboratory, Department of Civil Engineering, Ahmadu Bello University, Zaria, Nigeria.

$$\mathrm{f_{cylk}} = 0.85\mathrm{f_{ck}} - 1.6 \qquad (11)$$

where: $\mathrm{f_{cylk}}$ is characteristic cylindrical compressive strength and $\mathrm{f_{ck}}$ is characteristic cube compressive strength.

Development of Compressive Strength Predictive Models

The Nano Engineered Concrete (NEC) mixes characteristic cube compressive strength predictive models presented in Eqs. (12), (13) were developed with the aid of DataFit Statistical Package coded by Oakdale Engineering (2019) using the experimental data obtained. External data was not used to validate the models because they were derived to predict the characteristic cube compressive strengths obtained from the experimental work. Moreover, the aim of developing the models was to aid the reliability based sensitivity analysis of the nano engineered reinforced concrete beam where the effect of variability in mix design parameters is required. The first model predicts the characteristic cube compressive strength (f_{ck1}) of NEC as a function of Nanosilica Dosage (D_{NS}) in percentage and Cementitious Materials Content (C_T) in kg/m^3. However, the second model predicts the Characteristic Cube Compressive Strength (f_{ck2}) of NEC mixes as a function of Nanosilica Dosage(D_{NS}) in percentage and Water to Cementitious Materials Ratio (WC_R). The models are as under:

$$f_{ck1} = 6.512 + 5.263 \times 10^{-2} C_T + 5.924 D_{NS} - 2.529 D_{NS}^2 \qquad (12)$$

$$f_{ck2} = 63.561 - 76.136 WC_R + 5.924 D_{NS} - 2.529 D_{NS}^2 \qquad (13)$$

Determination of Optimal Nanosilica Dosage for the Concrete Grades

The study used the optimal nanosilica dasage of 1.5% and 1% by weight of cementations materials determined in Adamu *et al.* (2020a) and Adamu *et al.* (2020b) for grades 30 and 40 concrete mixes respectively. The mix notations for the optimal nano engineered concrete mixes for grades 30 and 40 concrete designated 30NS1.5 and 40NS1 respectively were also adopted from Adamu *et al.* (2020a) and Adamu *et al.* (2020b) and used in this work.

Derivation of Limit State Equations and Performance Functions

Conversion of Characteristics Cube to Cylindrical Compressive Strength

Eq. (14) was used to convert characteristic cube compressive strength values produced by the predictive models (Eqs. 12 and 13) to characteristic cylindrical compressive strength in line with the requirements of Eurocode 2 (EN1992-1-1: 2008). Eq. (14) was obtained from Domone (2010) but the symbols were modified to fit the developed compressive strength predictive models.

$$f_{cylk} = 0.85 f_{ck(1\ or\ 2)} - 1.6 \qquad (14)$$

where $f_{ck(1\ or\ 2)}$ and f_{cylk} are the respective characteristic cube compressive strengths and characteristic cylindrical compressive strengths derived from the predictive models.

Bending Criterion

The theory of bending indicates that for a section subjected to bending stresses as a result of transverse loading such as a simply supported beam, compressive stresses occur at the top fibre while the bottom fibre is subjected to tensile stresses. For a simply supported singly reinforced NEC beam having point load at mid span which was considered in this study, EN1992-1-1(2008) shows that the tensile and compressive capacities of the beam section can be determined using Eqs. (15) and (16) respectively.

$$M_c = 0.167 f_{cylk} b d^2 \qquad (15)$$

$$M_T = 0.87 f_{yk} A_s \left(d - \frac{0.87 f_{yk} A_s}{1.134 f_{cylk} b}\right) \qquad (16)$$

where f_{cylk} is the characteristic cylindrical compressive strength, b is the width of the section, d is the effective depth, A_s is the area of tension steel, f_{yk} is the characteristic steel strength, M_T is the tensile moment capacity and M_C is the compressive moment capacity.

The partial safety factors provided by EN 1992-1-1 (2008) to take care of uncertainties in concrete and reinforcing steel, which are 1.5 and 1.15 respectively were removed from Eqns. (15) and (16). This was to enable assessment of the actual safety provided in the design formulations. The equations were re-derived to yield and are given by Eqns. (17) and (18) for compressive and tensile bending respectively.

$$M_{RC} = 0.251 \phi_R f_{cylk} b d^2 \qquad (17)$$

where M_{RC}, f_{cylk}, d, b, and ϕ_R are the respective compressive moment of resistance, characteristic cylindrical compressive strength of concrete, effective depth of beam, beam breadth and resistance model uncertainty for compressive moment capacity.

$$M_{RT} = \phi_R f_{yk} \rho b d \left(d - \frac{f_{cylk} \rho b d}{1.7 f_{cylk} b}\right) \qquad (18)$$

where M_{RT}, f_{yk}, ρ, f_{cylk}, d, b and ϕ_R are the respective tensile moment of resistance, characteristic steel yield strength, reinforcement ratio, characteristic cylindrical compressive strength of concrete, effective depth of beam, beam breadth and resistance model uncertainty.

The maximum applied bending moment for a simply supported beam having point load at midspan and considering its self-weight was derived from basic structural analysis and is given by Eq. (19).

$$Ma = \phi_S (0.25 \mathrm{P} L + 0.125 \gamma_c b h g L^2 \times 10^{-9}) \qquad (19)$$

where M_a, P, γ, g, b, L, h and ϕ_S are the respective applied moment, applied load, unit weight of concrete, acceleration due to gravity, beam breadth, beam span, depth of beam and load model uncertainty.

The respective performance functions for compressive (G_{MC}) and tensile (G_{MT}) bending failure modes are given by Eqs. (20) and (21) based on Eqs. (2), (15), (18) and (19).

$$\boldsymbol{G_{MC}} = M_{RC} - M_a \qquad (20)$$

$$G_{MT} = M_{RT} - M_a \qquad (21)$$

where M_{RC}, M_{RT}, M_a are the respective compressive moment resistance, tensile moment resistance, and applied moment respectively.

Shear Criterion

The limit state equation for shear capacity of reinforced concrete sections without shear reinforcement provided by EN1992-1-1(2008) is as given by Eq. (22) after necessary substitutions in line with the code provisions.

$$V_c = \phi_R \left(0.18 \left(1 + \sqrt{\frac{200}{\mathrm{d}}}\right) \left(\rho f_{cylk}\right)^{1/3}\right) b d \qquad (22)$$

where Vc, $f_{cylk}, b, \rho, and\ \phi_R$ are the shear capacity, characteristic cylindrical compressive strength, beam width, tensile steel reinforcement ratio, and shear resistance factor respectively.

The maximum applied shear force derived from basic structural analysis for a simply supported beam having point load at midspan considering the effect of self-weight is given by Eq. (23):

$$V_a = \phi_S (0.5 \mathrm{P} + 0.5 \gamma_c b h \mathrm{L} g \times 10^{-9}) \qquad (23)$$

where Va P, g, γ_c, b, h are the maximum applied shear force, applied point load, acceleration due to gravity, unit weight of concrete, beam width and beam depth respectively.

The performance function (G_{vx}) for the shear criterion is given by Eq. (24) based on Eqs. (2), (22) and (23):

$$G_{vx} = V_c - V_a \qquad (24)$$

where Vc and Va are the shear capacity and applied shear force respectively.

Parameters of Stochastic Model

Table 4 presents the parameters of stochastic model used in the reliability sensitivity analysis. The mean values were obtained from the validated mix design, the coefficients of variation (COV) were derived from statistical analysis of experimental data, while the variables were considered to obey normal distribution as literature reveal that it can fit most design variable.

Table 4: Stochastic Parameters of Mix Design Variables

S/No	Basic Variable	Unit	Mean	COV	Distribution	Source(s)
1	Nano-Silica Dosage (D_{NS})	%	1.50	0.100	Normal	Distribution fits most variables
2	Total Binder Content (C_T)	%	433	0.100	Normal	Distribution fits most variables
3	Water Cement Ratio (WC_R)	%	0.45	0.100	Normal	Distribution fits most variables

Safety Index (β) Computations

The work utilized the concept of First Order Reliability Method (FORM) proposed by (Rackwitz, 1976; Rackwitz and Fiessler, 1978; Fiessler *et al.*, 1979) where the performance function G(X) was expanded using Tylor series expansion at some points on the failure surface not on the mean. The reliability computations using FORM normally considers each variable as a function of two moments (mean and standard deviation). The study employed FORM 5; which is a reliability software developed and coded in FORTRAN module by Gollwitzer *et al.* (1988) based on earlier works of (Rackwitz, 1976; Rackwitz and Fiessler, 1978; Hasofer and Lind, 1974). FORM5 was used by researchers (Abejide, 2014; Adamu, 2014; Abejide and Adamu, 2013; Wasiu and Adedeji, 2018) to carry out reliability analysis. Stochastic model parameters provided in Table 4 and the relevant performance functions for each failure mode were used. Programs were developed using FORTRAN Programming Language for the bending and shear failure modes and synchronized with FORM5 for the reliability analysis. Other parameters of the performance function not provided in Table 4 were considered deterministic with their design values fixed in the developed subroutines. The NEC mix design parameters were varied to observe the effect of variability under constant applied bending and shear stresses according to EN1992-1-1 (2008) and its reference standards. The results are presented in Figures 3 to 10.

RESULTS AND DISCUSSION

Properties of Preliminary and Validated Concrete Mixes

Table 5 presents the strength and workability properties of preliminary and validated concrete mixes for grade 30 and 40 concrete. The characteristic cube compressive strengths obtained for the preliminary concrete mixes were 24.25N/ mm^2 and 35.58 N/ mm^2 for grades 30 and 40 concrete respectively. These values indicate a shortfall of 23.71% and 12.42% from the desired characteristic cube compressive strength values of grades 30 and 40 concrete respectively. This is in line with the assertion of Neville and Brooks (2010) which says that concrete mix design provide an intelligent guess not the exact mix proportions for a given concrete grade. The failure of the designed concrete to meet the strength requirements of the concrete grades necessitates the need for adjustment of the mix proportions according to Neville and Brooks (2010). The strength and workability properties of the validated concrete mixes for the given concrete grades are also presented in Table 5. It could be observed from Table 5 that characteristic cube compressive strength values of 30.44N/ mm^2 and 40.83 N/ mm^2 were obtained for grades 30 and 40 concrete mixes respectively. The strength values obtained for the validated concrete mixes were 1.47% and 2.08% more than the target characteristic cube compressive strength of grades 30 and 40 concrete mixes respectively. The slump values obtained were considered adequate for production of NEC whose high surface area normally reduces the workability of concrete. Therefore, it could be said that the validated concrete mix parameters for the two concrete grades were adequate for the production of NEC.

Table 5: Properties of Preliminary and Validated Concrete Mixes

Mix Type	Concrete Grade	Strength Parameters		Slump		Statistics	
		Fck N/mm^2	Fm N/mm^2	Design mm	Achieved mm	SD	COV
Preliminary	30	24.25	27.67	80	155	2.08	0.08
	40	35.58	36.37	80	165	0.58	0.02
Validated	30	30.44	32.33	80	135	1.16	0.04
	40	40.83	43.33	80	130	0.58	0.02

Predictive Models for Characteristic Compressive Strength of NEC

The 3D plots of predicted and experimental characteristic cube compressive strength are presented in Figures 1 and 2. The model presented in Figure 1 gives characteristic cube compressive strength of NEC as a function of nanosilica dosage (D_{NS}) and cementitious materials content (C_T). Similarly, Figure 2 presents characteristic cube compressive strength of NEC as a function of nanosilica dosage (D_{NS}) and water to cementitious materials ratio (W_{CR}). Since the models were developed for the purpose of reliability analysis, validation was only made against the data points used in the models development. In the plots, mesh and dots were used to depict the predicted and experimental data points respectively. Equations 12 and 13 were represented by plots in Figures 1 and 2 respectively. The Coefficients of Multiple Determination (R^2) reported by DataFit statistical package for plots in Figure 1 (Eq. 12) and Figure 2 (Eq. 13) were 0.9474 and 0.9394 respectively. This signifies that the model explained 94.74 % and 93.94 % of the variation in characteristic cube compressive strength of NEC. This shows that nanosilica dosage (D_{NS}) and cementitious materials content (C_T) explained 94.74 % of the variation in NEC characteristic cube compressive strength presented in Eq. 12. In the same way, nanosilica dosage (D_{NS}) and water to cementitious materials ratio (W_{CR}) explained 93.94 % of the variation in NEC characteristic cube compressive strength presented in Eq. 13. The values of Coefficients of Multiple Determination (R^2) obtained indicate that the models developed can adequately predict the characteristic cube compressive strength of NEC. It could be deduced from Figures 1 and 2 that the characteristic cube compressive strength of NEC increase with increase in cementitious materials content and nanosilica dosage within the range of 0.5% to 1.5% but drops when nanosilica dosage exceeds the optimal value of 1.5% by weight of cementitious materials in line with experimental outcomes (Adamu *et al.*, 2020a). However, the characteristic cube compressive strength of NEC decrease with increase in water to cementitious materials ratio in line with the experimental outcomes as shown in Figure 2. From the discussion, it could be concluded that the developed characteristic cube compressive strength predictive models can adequately predict the experimental data points used in the models development.

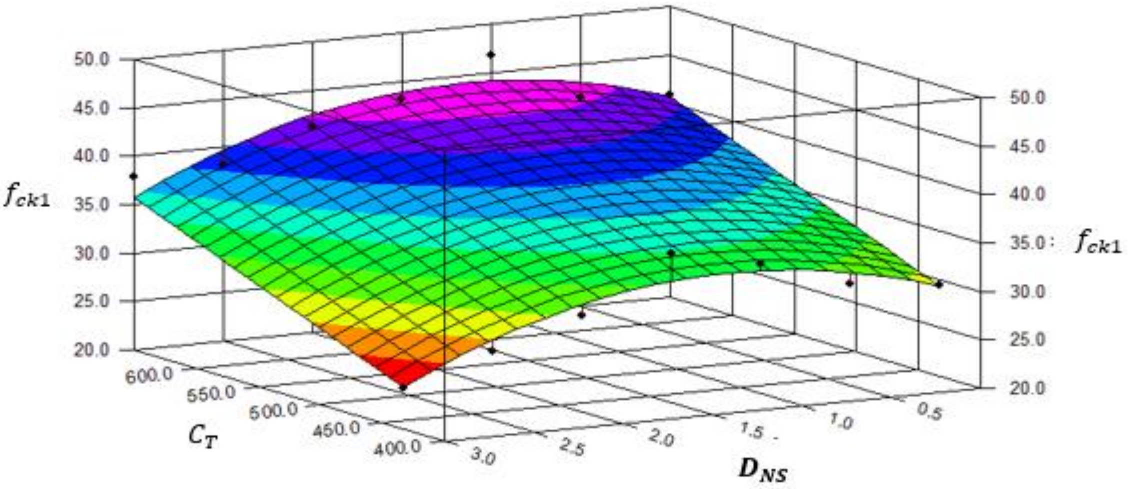

Figure 1: Experimental and Predicted Data Points for Equation 12

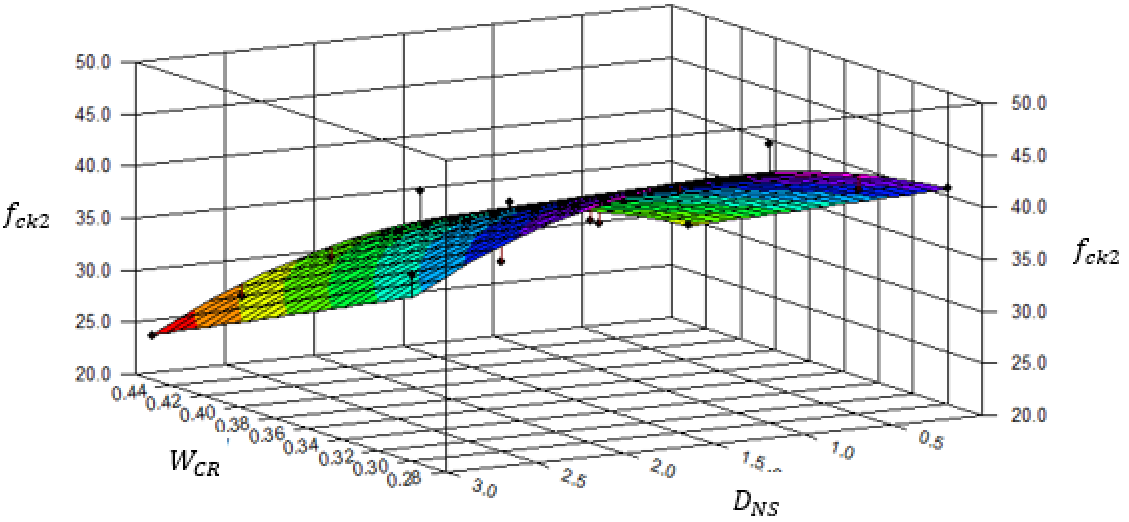

Figure 2: Experimental and Predicted Data Points for Equation 13

Effect of Nanosilica Dosage on Safety of NERC Beam

Figures 3, 4 and 5 present the respective variation of safety index (β) with change in Nanosilica (nS) dosage for Nano Engineered Reinforced Concrete (NERC) beam under compressive bending, tensile bending and shear failure modes. It is apparent from Figures 3, 4 and 5 and for 30NS1.5 optimal NEC mix that safety index increase with increase in nS dosage in the range of 0.50 % to 1.50 % by weight of cementitious materials. Similarly, an increase in safety index could be observed from these figures when nS dosage was increased in the range of 0.50 % to 1.00 % by weight of cementitious materials for 40NS1 mix. A decrease in safety index was observed from Figures 3, 4 and 5 when nS was increased beyond the respective Optimal Nanosilica Dosages (ONSD) of 1.5% and 1% by weight of cementitious materials for 30NS1.5 and 40NS1 NEC mixes, which agrees with experimental results reported in Adamu *et al.* (2020a) and Adamu *et al.* (2020b). The decrease in safety index with increase in nS dosage beyond ONSD could be attributed to agglomeration in the NEC mixes, whose effect reduces the characteristic cylindrical compressive strength of the NERC beam. Safety index decreases in the respective range of 2.27, 2.10 and 0.83 for compressive bending, tensile bending and shear failure modes when nS dosage was increased from 1.5% (ONSD) to 4% by weight of cementitious materials considering 30NS1.5 mix. However, the range of safety index loss for compressive bending, tensile bending and shear failure modes were 1.39, 0.20 and 0.47 respectively when nS dosage was increased from 1% (ONSD) to 4% by weight of cementitious materials for 40NS1 mix. Therefore, it could be deduced from the ranges that excess nS dosage can have the most pronounced effect in reducing NERC beam safety for the three failure modes in the order of compressive bending, tensile bending, and then shear. The ranges also indicate that an increase in characteristic cylindrical compressive strength caused by increase in NEC grade (from 30NS1 to 40NS1) can lead to reduction in the effect of excess nS dosage on the NERC beam safety for the three failure modes. Therefore, it could be concluded that nS dosage in NERC beams should not exceed optimal value if a positive effect in beam safety is required.

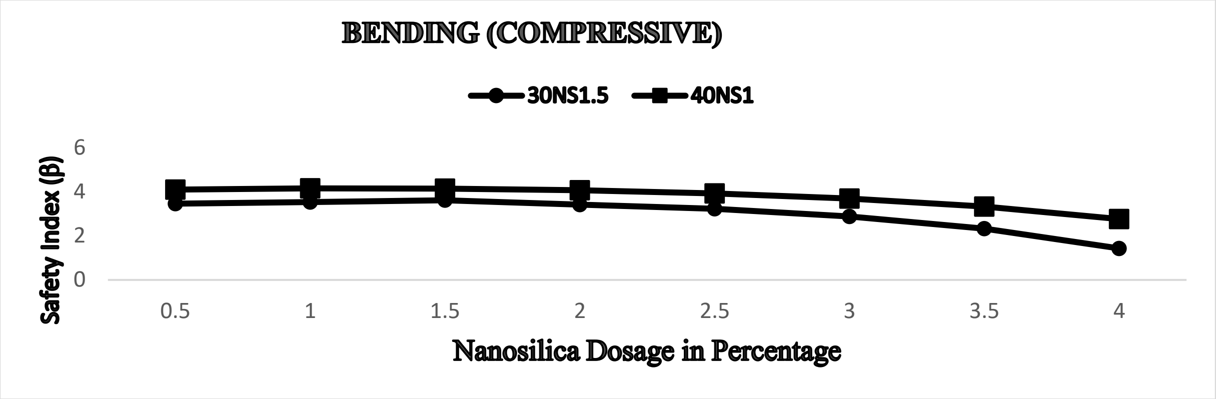

Figure 3: Effect of nanosilica dosage on safety of NERC beam under compressive bending

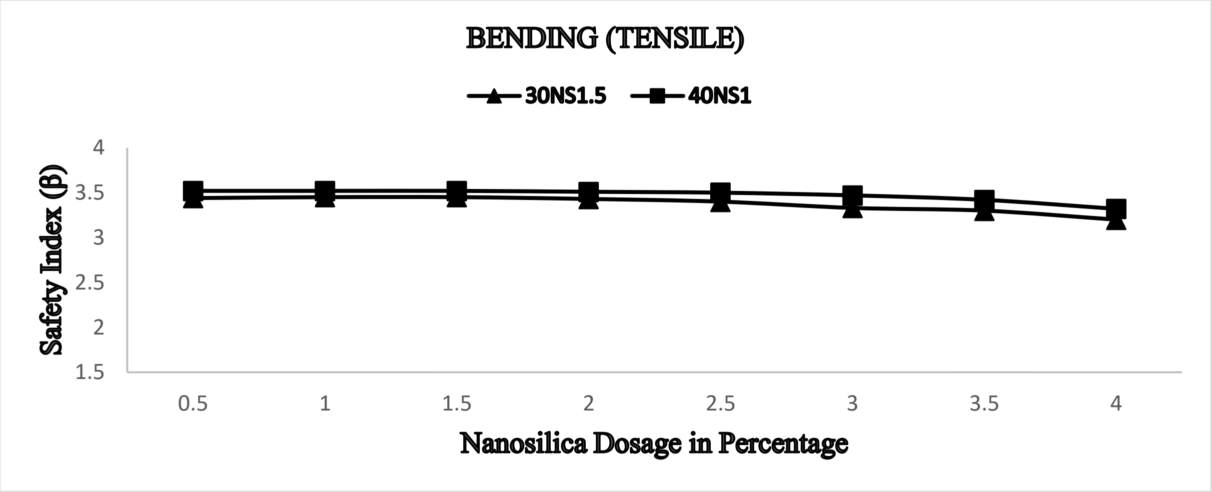

Figure 4: Effect of nanosilica dosage on safety of NERC beam under tensile bending

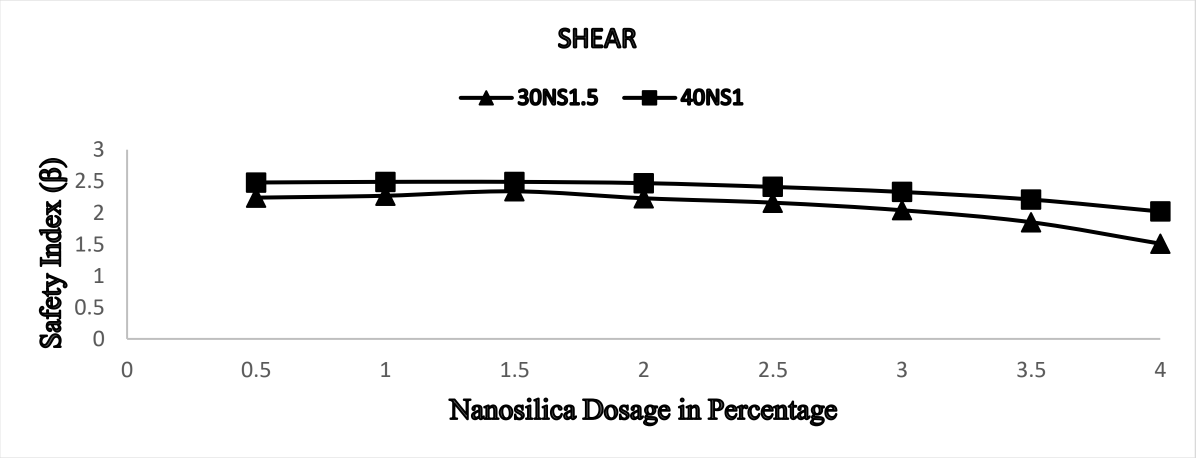

Figure 5: Effect of nanosilica dosage on safety of NERC beam in shear

Effect of Cementitious Materials Content on Safety of NERC Beam

The effect(s) of varying cementitious materials (total binder) content on the safety of NERC beam under compressive, tensile bending and shear failure modes considering the two optimal nano engineered concrete mixes (30NS1.5 and 40NS1) is presented in Figures 6, 7 and 8 respectively. In Figures 6, 7 and 8, an increase in safety of the NERC beam was observed when cementitious materials content was increased irrespective of the NEC mix considered. From the safety index values obtained, it is obvious that variation in the grade of NEC mix (from 30NS1.5 to 40NS1) has no effect on the NERC beam safety. This could be due to the fact that a cement type exhibits the same pattern of strength development. Variation of cementitious material content from 50 kg/m^3 to 1000 kg/m^3 led to gain in safety of NERC beam in the range of 3.35, 1.02 and 1.23 for compressive bending, tensile bending and shear failure modes. This indicates that the order of positive contribution in NERC beam safety due to increase in cementitious materials content is from compressive bending to shear and then to tensile bending failure modes. This could be due to the fact that characteristic cylindrical compressive strength contributes in the computation of shear and compressive bending capacity of beams, but does not add value in the computation of tensile capacity of beams according to EN1992-1-1 (2008) and its reference standards. The observed increase in safety of NERC beam with increase in cementitious materials content under tensile bending could be attributed to the contribution of cementitious materials content in varying NERC beam design variables, as a result of its positive effect on compressive strength development of NERC beams. Therefore, it could be concluded that compressive strength contribute to tensile strength of NERC beams through composite action.

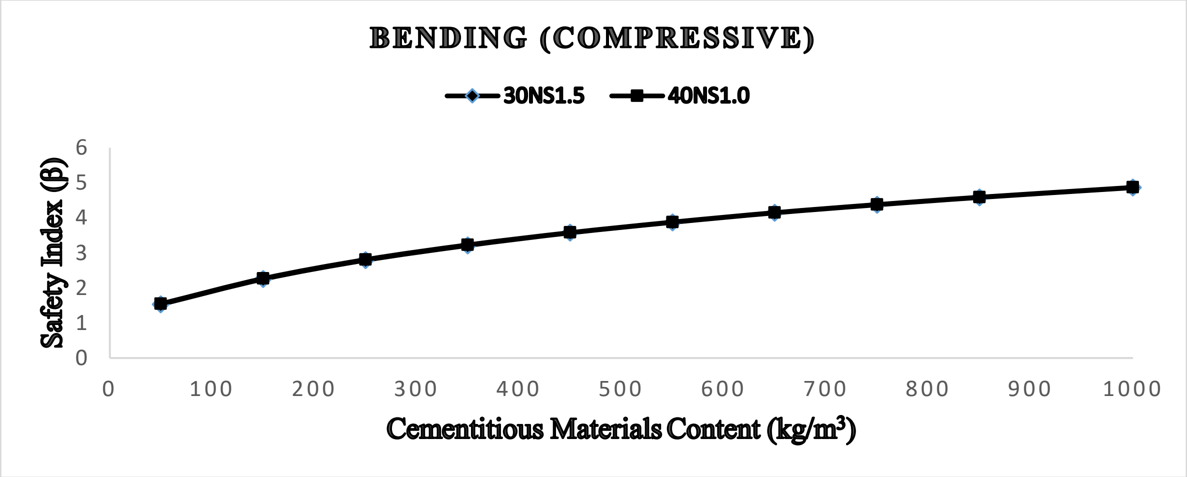

Figure 6: Effect of cementitious materials content on NERC beam safety in compressive bending

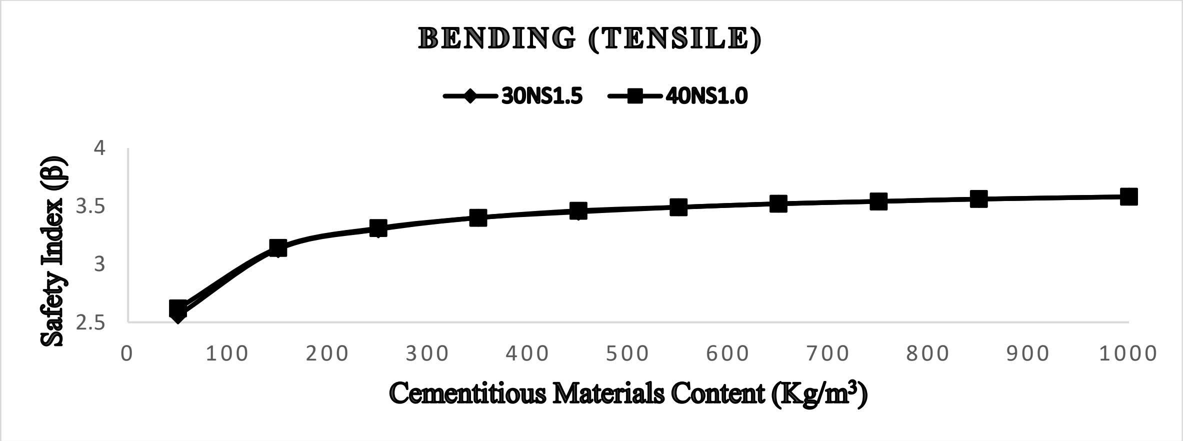

Figure 7: Effect of cementitious materials content on NERC beam safety in tensile bending

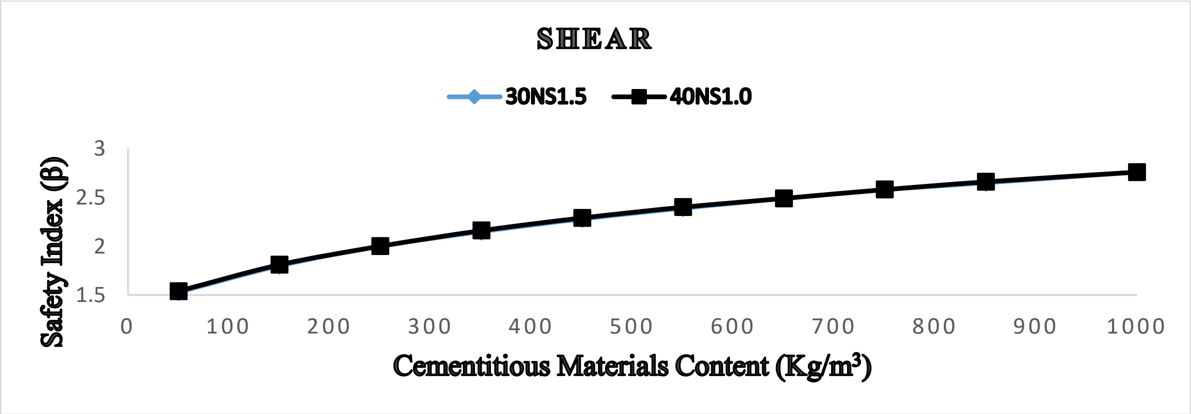

Figure 8: Effect of cementitious materials content on NERC beam safety in Shear

Effect of Water to Cementitious Materials Ratio on NERC Beam Safety

Figures 9 and 10 present the effect of varying water to cementitious materials ratio on Nano Engineered Reinforced Concrete (NERC) beam safety under compressive bending and shear failure modes and for the optimal NEC mixes (30NS1.5 and 40NS1). Generally, the trend indicates that decrease in water to cementitious materials ratio resulted to increase in the safety of NERC beam due to increase in characteristic cylindrical compressive strength of NEC in line with Abraham’s water to cementitious materials ratio law. The range of safety index loss in compressive bending and shear were 4.94 and 1.89 respectively when water to cementitious materials ratio of the NEC mixes was varied from 0.05 to 0.80. The results show that the negative effect of variation in water to cementitious materials ratio on safety of NERC beam is more pronounced in compressive bending than under shear failure modes. The reduction in safety of NERC beam as a result of increase in water to cementitious materials ratio can be attributed to loss in characteristic compressive strength of NEC due to reduced densification as a result of inappropriate addition of water. Therefore, it is imperative to produce a NEC mix that combines appropriate amount of water with cementitious materials content for improved safety of NERC beams subjected to bending and shear stresses.

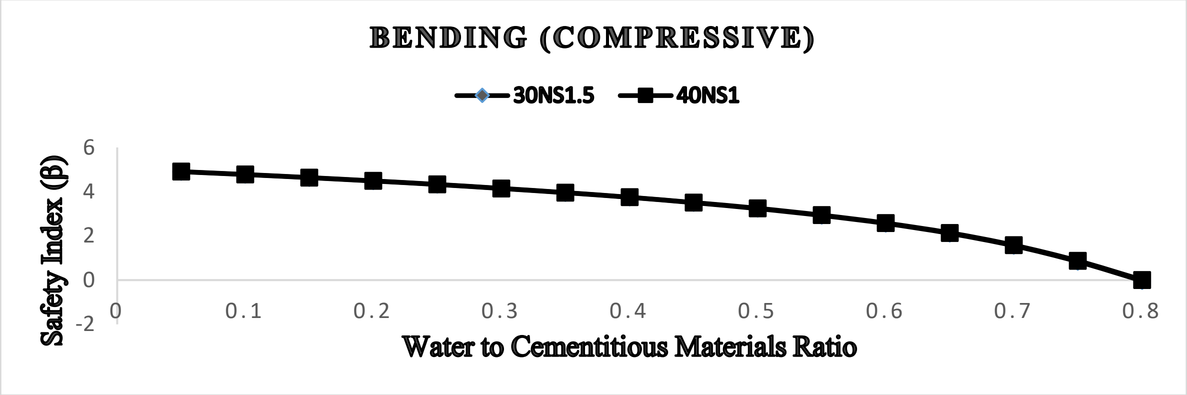

Figure 9: Effect of water to cementitious materials ratio on safety of NERC beam in compressive bending

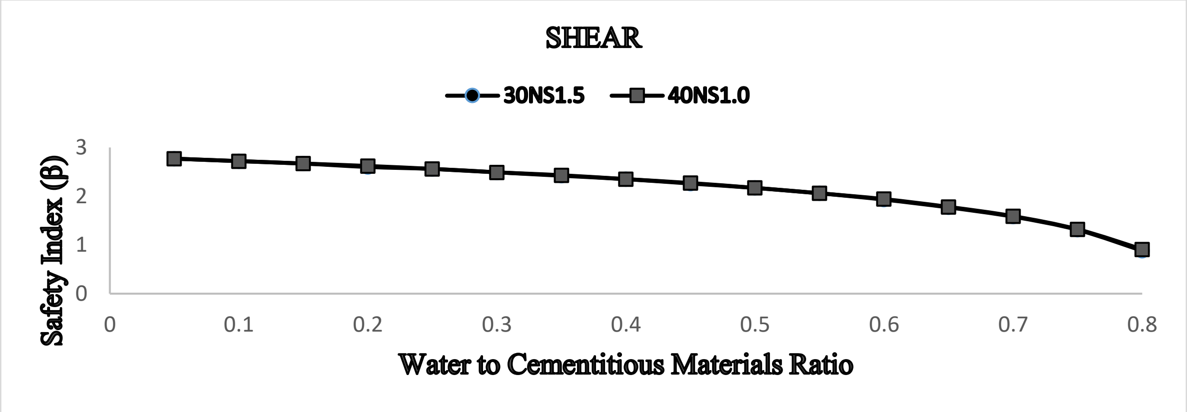

Figure 10: Effect of water to cementitious materials ratio on safety of NERC beam in shear

CONCLUSIONS

This paper presents the effect of variation in Nano Engineered Concrete (NEC) mix design parameters on structural safety of Nano Engineered Reinforced Concrete (NERC) beam determined using reliability based sensitivity analysis in line with the provisions of EN1992-1-1 (2008) and its reference standards. In the study, characteristic compressive strength predictive models for NEC were developed and validated using the experimental data obtained. The results show that variation in NEC mix design parameters affect the structural safety of NERC beams whose effect is determined by how the NEC mix design parameter contributes to characteristic compressive strength development in NEC. Increase in nanosilica dosage beyond optimal value (ONSD) was found to have negative effect on the beam safety while decrease in water to cementitious materials ratio led to increase in structural safety of the NERC beam. Additionally, increase in cementitious materials content was found to have positive effect on structural safety of the NERC beam. Contrary to assumptions made in Reinforced Concrete Design (RCD) that concrete does not contribute to tensile strength capacity of beams, the reliability sensitivity analysis shows that NEC contributes to the tensile strength of beams through composite action. Moreover, the models developed were found to be capable of predicting the characteristic compressive strength of NEC. The study shows that reliability concepts are essential in predicting the safety level of NERC beams where variability in mix design parameters cannot be avoided at implementation in natural situations, as deterministic approach incorporated in codes of practice cannot be adequate. Furthermore, the study suggested that the effect of variability in concrete mix design parameters be incorporated in design formulations of Civil Engineering Codes of Practice to assist in estimation of safety of structural elements to be designed and produced with NEC for improved safety, sustainability and resilience of Civil Engineering infrastructure.

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