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AN EOQ MODEL FOR ITEMS THAT ARE BOTH AMELIORATING AND DETERIORATING WITH LINEAR INVENTORY LEVEL DEPENDENT DEMAND

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ABSTRACT

It is normally observed in real life that the volume of stocked items have motivational effect on customers. Stores with large amount of displayed items attract buyers more than stores with scanty items. In this study therefore, we develop an EOQ model for linear inventory level dependent demand for items that undergo both deterioration and amelioration at the same time while at inventory level with no shortage of items. The poultry, fishery, piggery, and so on where the stock increase in weight and/or value within a short time and decrease in volume due to diseases or other factors provide good example. The model determines the best cycle length so as to minimize the overall cost. Numerical examples are given to illustrate the model and a sensitivity analysis carried out to see the effect of changes to some model parameters on the decision variables.

Keywords: EOQ, Amelioration, Deterioration, Partial backlogging, Linear level inventory demand

INTRODUCTION

The decay that prevents items from being used for their intended purpose is termed deterioration. Amelioration on the other hand, refers to a situation where stocked items increase in quantity and/or quality while in stock. Some items have the property of incurring amelioration and deterioration albeit simultaneously as they are kept in warehouse. A lot of research has been carried out over the years on controlling the inventory of items having one or both of the properties above. The traditional Economic Order Quantity (EOQ) Model assumed that the demand for stocked items remains naturally steady for all time to come. In reality however, this assumption is unrealistic as demand may be influenced by the amount of items held in stock, the passion-ability trend, the freshness of the items, the selling price and so on.

Not all items deteriorate when in stock as some of them undergo increase in quantity or quality or both. Generally, fast growing animals like fishes, poultry, cattle, etc, provide good examples. Some fruit merchants in some tropical countries invest huge amount of money in buying large plantations of orange, banana, pineapple, etc and stock them for months waiting for the arrival of times of festivities when the demand shoot up geometrically. It is obvious that within this period, the items gain value and/or quantity. The items with such properties are referred to as *ameliorating items.*

The existing literature on inventory gives little attention to the ameliorative nature of inventory. It was not until in the late 90's that Hwang (1997), for the first time studied an economic order quantity (EOQ) model and a partial selling quantity (PSQ) model in connection with ameliorating items under the assumption that the ameliorating time follows the Weibull distribution. Again, Hwang (1999, 2004) developed inventory models for both ameliorating and deteriorating items separately under the LIFO and FIFO issuing policies. Later, Moon *et al.* (2005, 2006) developed an EOQ model for ameliorating/deteriorating items under inflation and time discounting. The model studied inventory models with zero-ending inventory for fixed order intervals over a finite planning horizon allowing shortages in all but in the last cycle. They also developed another model with shortages in all cycles taking into account the effects of inflation and time value of money. Later, Mondal *et al.* (2005) developed a partial selling inventory model for ameliorating items under profit maximization.

The consumption rate of certain items depends on the amount of on-hand inventory. Levin *et al.* (1972) demonstrated that large stockpile of inventory in store motivates buyers to purchase more. These and similar observations have attracted many marketing researchers and practitioners to investigate the modeling aspects of this phenomenon. Gupta and Baker (1986) studied a model that minimized the general inventory cost with the assumption that the consumption rate is dependent on the initial stock level. Urban (1988) have earlier focused on the analysis of inventory systems which describe the demand rate as a power function, dependent on the level of the on-hand inventory.

Valliathal (2016) studied the effects of inflation and time discounting on an EOQ model for time dependent deteriorating/ameliorating items with general ramp type of demand and partial backlogging rate. Valliathal (2016) studied the model under the replenishment policy, starting with shortages under two different types of backlogging rates and provided the comparative analysis.

Srivastava (2017) developed an inventory model for ameliorating/deteriorating items with trapezoidal demand and complete backlogging under inflation and time discounting. Srivastava (2017) proposed an inventory model for ameliorating/deteriorating items with inflationary condition and time discounting rate and also completely backlogged shortage.

Vandana (2019) has carried out a research work titled, "A two-echelon inventory model for ameliorating /deteriorating items with single vendor and multi-buyers'' where a model that proposes a fixed period for buyers and reduces the integrated total cost of the inventory was considered. The model discussed a case of single manufacturer who produces the ameliorating items and sells the finished goods to the multiple buyers.

An economic order quantity model for ameliorating inventory where the lead time, the replenishment time and the demand rate are constants with no shortage of items was studied by Gwanda and Sani (2011). The model obtained an optimum ordering quantity while keeping the relevant inventory costs minimum. Again Gwanda and Sani (2012) extended their earlier model to allow for linear trended demand.

For most stocked items the amount maintained normally depends on the rate at which the item is consumed and the rate of its consumption is observed by many researchers, to in turn depend on the volume of the inventory stocked. The consumption rate fluctuates with the on-hand stock level and hence large sized stores are observed to attract more customers than smaller ones. This property is also evident in most ameliorating inventories, like fishes, poultry, husbandry, and so on. In all these kinds of inventory, the demand tends to increase with increase in the volume of stock. It is a common knowledge that stores with larger stocks have more appeal to customers as both the quantitative and qualitative tastes are more likely to be met therein. When the store runs out of stock however, customers normally place backorders and wait for resupply. Taking longer time without receiving the supply may tempt some customers to go elsewhere resulting in lost sales.

In all the models above, the unsatisfied demand (whenever there are shortages) was assumed to be completely backlogged. In many cases however, demand for items is lost during the shortage period. Montgomery *et al.* (1973) studied both the deterministic and stochastic demand inventory models with a mixture of backorders and lost sales. Later, Rosenberg (1979) provided a new analysis of partial backorders. Mark (1987) modified the model of Rosenberg (1979) by incorporating a uniform replenishment rate to determine the optimal production-inventory control policies.

An inventory model for non-instantaneous deteriorating items with stock-dependent demand was developed by Wu et al (2006) where they considered a linearly stock dependent demand and a constant unit holding cost. In the model, shortages are allowed and the backlogging rate is a variable and dependent on the waiting time for the next replenishment. Chang *et al.* (2010) amended Wu *et al.* (2006)'s model by changing the objective to maximizing the total profit and setting a maximum inventory level in the model to reflect the facts that most retail outlets have limited shelf space. Chang *et al.* (2010) also relaxed the restriction of zero ending inventory when shortages are not desirable and provided an algorithm to find the optimal solution.

An inventory model under inflation for deteriorating items with stock-dependent consumption rate and partial backlogging shortages was developed by Yang *et al.* (2010) by considering an inventory lot-size model under inflation for deteriorating items with stock-dependent consumption rate when shortages are partially backlogged.

Shukla and Tripathi (2017) presented an EOQ model with stock-level dependent and different demand and deferent types of holding cost level. The authors showed that the total relevant inventory cost per unit time is convex with respect to cycle period and they determined the optimal order quantity and relevant cycle cost

An economic order quantity (EOQ) inventory model with stock dependent demand under multiple cases of permissible delay for deteriorating items has been derived by Handa *et al.* (2020) where the demand rate is stock dependent with shortage which is partially backlogged and the backlogging rate is exponentially decreasing. Handa *et al.* (2020) determined the convexity of the optimal solution for different cases of trade credit. The model was exemplified numerically for different cases. The impact of demand parameter, deterioration rate, backlogging parameter and interest earn rate on critical time as well as on total system cost were demonstrated by sensitivity analysis.

Mallick *et al.* (2021) studied an EOQ model for breakable items where the demand of the business is stock dependent. The breakability of the items is dependent on stock and holding cost and a credit period offered to the retailer depending on lead time. Stock dependent demand and breakability are balanced by the lead time dependent credit period. The model detects the optimal order quantity, optimal lead time and maximizes the total profit for the stockiest.

Zhang *et al.* (2022), investigated the retailer's strategy in selecting the order-up-to level, the reorder point and the preservation technology investment for deteriorating items, aiming to maximize total profit per unit time. The model was formulated taking into account the stock-dependent demand rate and stock-dependent holding cost. The authors relaxed the terminal conditions showing that the reorder point can be one of the following two cases:

i. $ N\leq0 $, where the reorder point may be negative or zero. When it is negative, the shortage occurred and is partially backlogged.
ii. $ N\geq0 $, when the reorder point has no shortage or it is zero.

Zhang *et al.* (2022) proved the existence and uniqueness of the optimal order-up-to level, the reorder point and the preservation technology investment under any given two cases and presented an algorithm to search for decision variables such that the total profit per unit time is maximized.

The linearity or otherwise of stock dependent demand was also a subject of intensive research. Cárdenas-Barrón (2020) for instance, studied an EOQ model with nonlinear stock dependent holding cost, nonlinear stock dependent demand and trade credit. The authors developed the model from retailers' point of view where the supplier offers a trade credit period to the retailer. The model relaxed the traditional assumption of zero ending inventory level to a non zero ending which was consequently obtained as positive, zero or negative.

In this paper, we develop an economic quantity model for items that are simultaneously ameliorating and deteriorating with stock dependent demand and partial backlogging. The model determines the optimum cycle length so as to keep the overall costs minimum.

The proposed inventory model is developed under the following assumptions and notation:

Assumptions

i. The inventory system involves only one single item and one stocking point.
ii. Amelioration and deterioration occur when the items are effectively in stock.
iii. The stock dependent demand rate $R(t)$ at time t is assumed to be $R[(t)] = \gamma + \tau V(t)$, where γ is a positive constant, τ is the stock dependent demand rate parameter, $0 < \tau < 1$, and $V(t)$ is the non-negative inventory level at time t.

Notation

- The cycle length is T.
- The length of time when the inventory start running into shortage is T_1.
- The inventory carrying cost in a cycle is H_c
- The unit cost of the item is a known constant C.
- The replenishment cost is also a known constant O_c per replenishment.
- Inventory holding charge per unit i, is a known constant.
- The level of inventory at any time t is $V(t)$.

- The initial inventory is what enters into the inventory at t = 0, and it is given by V_0 .
- The amount of inventory in the interval (0,T) is V_T
- The rate of amelioration a is a constant.
- The rate of deterioration d is a constant
- The ameliorated amount over the cycle T when considered in terms of value (say, weight) is given by a_T .
- The total number of deteriorated units in a cycle when considered in terms of value is d_T .

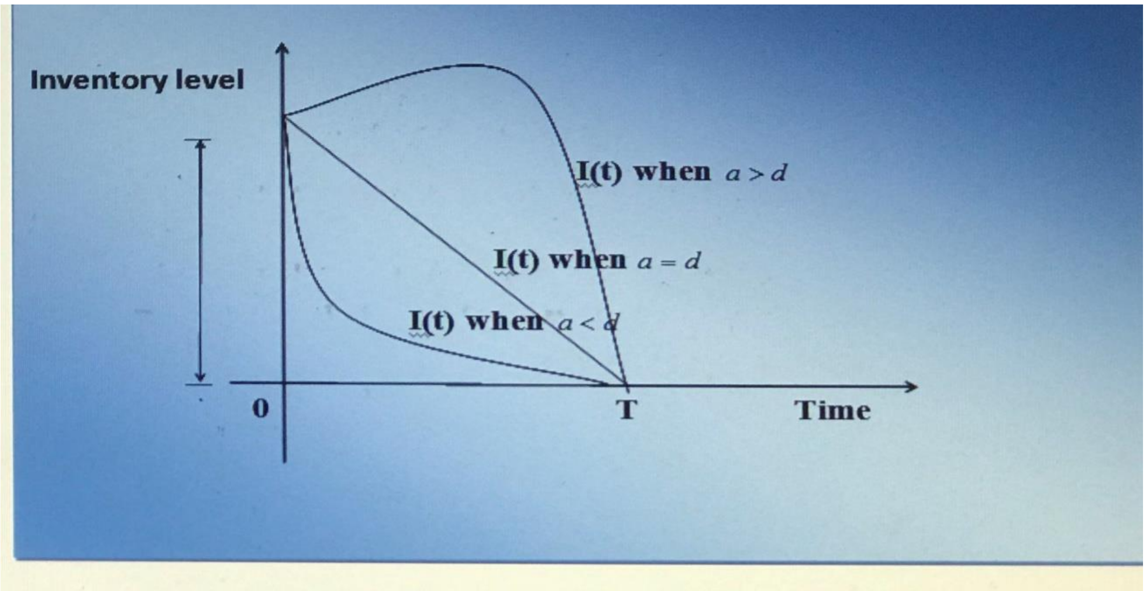

Figure 1: The graphical representation for the inventory system

METHODOLOGY

Model formulation

Our objective is to determine the optimal replenishment time such that the total relevant inventory costs are kept at a minimum. Let $V(t)$ be the volume of the on hand inventory at time $t \geq 0,$ then at time $t + \Delta t,$ the on hand inventory in the interval $(0, T_1)$ is given by:

$$V(t + \Delta t) = V(t) + a.V(t)\Delta t - d.V(t)\Delta t - (\gamma + \tau V(t)).\Delta t$$

Dividing by Δt and taking limit as $\Delta t \to 0,$ we obtain;

$$V'(t) + (d - a + \tau)V(t) = -\gamma, \qquad 0 \leq t \leq T \tag{1}$$

The solution of equation (1) is obtained as;

$$V(t) = -\frac{\gamma}{d - a + \tau} + ke^{-(d-a+\tau)t}, \text{ where } k_1 \text{ is a constant.} \tag{2}$$

Using $V(0) = V_0$, we obtain the value of k as:

$$k = V_0 + \frac{\gamma}{d - a + \tau}, \tag{3}$$

The value of k is then substituted in equation (2) to get;

$$V(t) = -\frac{\gamma}{d - a + \tau} + \left[V_0 + \frac{\gamma}{d - a + \tau}\right]e^{-(d-a+\tau)t}$$

$$= -\frac{\gamma}{d - a + \tau} + \frac{\gamma}{d - a + \tau}e^{-(d-a+\tau)t} + V_0 e^{-(d-a+\tau)t} \tag{4}$$

Applying the boundary condition $V(T) = 0,$ we get:

$$\Rightarrow V_0 = \frac{\gamma}{d - a + \tau}(e^{(d-a+\tau)T} - 1) \tag{5}$$

The value of V_0 is substituted into equation (4) to obtain:

This gives; $V(t) = \frac{\gamma}{d-a+\tau}\left(e^{(d-a+\tau)(T-t)} - 1\right)$ (6)

Total Amount of on Hand Inventory during the Complete Cycle Time T

This is given by:

$$V_T = \int_0^T V(t)dt = \frac{\gamma}{(d-a+\tau)^2}(e^{(d-a+\tau)T} - \mathrm{T}(d-a+\tau)-1) \quad (7)$$

The deteriorated amounts in (0, T) is:

$$d_T = dV_T \quad = \frac{d\gamma}{(d-a+\tau)^2}(e^{(d-a+\tau)T} - \mathrm{T}(d-a+\tau)-1) \quad (8)$$

The ameliorated amount over the cycle T is given by:

$$a_T = a\mathrm{V}_T \quad = \frac{a\gamma}{(d-a+\tau)^2}(e^{(d-a+\tau)T} - \mathrm{T}(d-a+\tau)-1) \quad (9)$$

The inventory holding cost in a cycle is obtained as:

$$H_c = iCV_T \quad = \frac{iC\gamma}{(d-a+\tau)^2}(e^{(d-a+\tau)T} - \mathrm{T}(d-a+\tau)-1) \quad (10)$$

Total variable cost per unit time

This is obtained as:

$T_c(T) = \frac{1}{T}$ {Ordering cost + Inventory holding cost per cycle + the deterioration cost per cycle – the amelioration cost per cycle }

$$\therefore T_c(T) = \frac{1}{T}\{O_c + iCV_T + CdV_T - CaV_T\}$$

$$= \frac{1}{T}\{O_c + C(i+d-a)V_T\}$$

$$= \frac{O_c}{T} + \frac{C\gamma(i+d-a)}{(d-a+\tau)^2 T}(e^{(d-a+\tau)T} - \mathrm{T}(d-a+\tau)-1) \quad (11)$$

$$\frac{d(T_c(T))}{dT} = \frac{d}{dt}\left(\frac{O_c}{T}\right) + \frac{d}{dt}\left(\frac{C\gamma(i+d-a)}{(d-a+\tau)^2 T}(e^{(d-a+\tau)T} - \mathrm{T}(d-a+\tau)-1)\right)$$

$$= -\frac{O_c}{T^2} + \frac{C\gamma(i+d-a)}{(d-a+\tau)^2 T^2}\left[(T(d-a+\tau)-1)e^{(d-a+\tau)T} + 1\right]$$

For optimal cycle period $\frac{d(T_c(T))}{dT} = 0$, that is,

$$0 = -\frac{O_c}{T^2} + \frac{C\gamma(i+d-a)}{(d-a+\tau)^2 T^2}\left[(T(d-a+\gamma)-1)e^{(d-a+\tau)T} + 1\right]$$ which simplifies to.

$$0 = -(d-a+\tau)^2 O_c + C\gamma(i+d-a)\left[(T(d-a+\tau)-1)e^{(d-a+\tau)T} + 1\right] \quad (12)$$

$$EOQ = V_0 = \frac{\gamma}{d-a+\tau}(e^{(d-a+\tau)T} - 1) \quad (13)$$

Equation (12) can then be solved to obtain the optimum values T^* of T using any suitable numerical method provided that, $\frac{\partial^2[T_c(T^*)]}{\partial T^2} > 0$,

That is, $\frac{2O_c}{T^3} + \frac{C\gamma}{(d-a+\tau)^2}\left[T(d-a+\tau)-(d-a+\tau)+1\right]e^{(d-a+\tau)T} > 0$ (14)

Newton-Raphson method for instance could be employed to solve the equation and obtain a solution for T. These solutions T^* gives the optimal solution of (12) provided equation (14) is true.

Numerical Examples

Equation (12) is used to obtain the solution of the following numerical example;

Input Parameters; O_c = 1000, C = 222, a = 0.6, d = 0.4, τ = 0.42, i = 0.4, γ = 10000

Output Parameters; T* = 25 days, EOQ = 2993859 Units, TVC(T)*= 29882 naira

Sensitivity Analysis

Next, we carry out a sensitivity analysis to see the effect of parameter changes on the decision variables. This has been carried out by changing (that is, increasing or decreasing) the parameters by 1%, 5%, and 25% and taking one parameter at a time, keeping the remaining parameters at their original values. The results are as given in Table 1

Table 1: Sensitivity analysis of the given example to see the effect of parameter changes

Parameter	% change in parameter value	% change in the value of the decision variables		
		T^*	$TVC(T)^*$	EOQ^*
O_c	-25	-25	-13	14
	-5	-5	-2	-9
	-1	-1	0	-9
	1	1	1	-4
	5	5	3	-4
	25	25	12	-11
C	-25	16	-13	-14
	-5	4	-2	-4
	-1	4	0	-4
	1	0	0	0
	5	0	2	0
	25	--8	11	9
a	-25	-24	32	-54
	-5	-4	7	-19
	-1	0	2	-5
	1	4	-1	2
	5	8	-9	24
	25	100	-50	395
d	-25	44	-29	134
	-5	8	-5	12
	-1	4	-1	0
	1	0	1	-4
	5	-9	5	-13
	25	-16	23	-44
τ	-25	-0	0	267
	-5	0	0	22
	-1	0	0	4
	1	0	0	-4
	5	0	0	-17
	25	0	0	-54
γ	-25	16	-13	-35
	-5	4	-2	-9
	-1	4	0	-5
	1	0	1	1
	5	0	3	5
	25	-8	12	36
i	-25	40	-29	-29
	-5	8	-5	-7
	-1	4	-1	-4
	1	0	-1	-9
	5	-4	5	4
	25	-16	22	19

DISCUSSION

Table 1 clearly shows that decision variables have strong effect on changes in the values of the parameters. The decision variables are sensitive to changes in all the parameters except τ in the case of T* and TVC(T)*. We also notice the followings from the table:

i. The parameters O_c, and a have linear relationship with cycle period while the parameters C, d, γ, and i have an inverse relation with the optimal cycle period T.

ii. The parameters C, a, γ and i have linear relationship with optimal EOQ while the parameters O_c, d, and τ have an inverse relation with the optimal EOQ.

iii. The parameters O_c, C, d, γ, and i have linear relationship with average total cost while the parameter a has an inverse relation with the optimal average total cost TVC(T).

The Table above shows that T* and TVC(T)* increase with increase in ordering cost. This is expected since if the ordering

cost increases, the total cost $TVC(T_1^*, T^*)$ will increase and the frequency of orders will reduce so as to reduce the cost and this in effect reduces the order quantity EOQ*.
From the table also, one sees that as expected, the increase in the item's cost results in increase in the EOQ and total variable cost which will invariably result in a decreased cycle period as the stockiest may not be able to stock plenty due to the prohibitive cost.
The model has provided us with interesting scenario involving its ameliorative and deteriorative behavior where it conforms to the common expectation that amelioration and deterioration go in opposite direction. We notice from the table that as the rate of amelioration α, increases, the ordering quantity EOQ^* also increases resulting in higher values of the cycle period T^*. On the other hand, as the rate of deterioration, d increases, the ordering quantity EOQ^* and the cycle period decrease resulting in higher value of the two TVC(T)*.

CONCLUSION

In this paper an economic order quantity model for both ameliorating and deteriorating items in which the demand rate is linearly dependent on inventory level has been presented. The model determines the optimal quantity to order while keeping the relevant inventory costs minimum. Numerical examples are given to illustrate the developed model and sensitivity analysis carried out on the results obtained from one of the examples in order to see the effect of parameter changes on the decision variables. The sensitivity analysis shows that all the decision variables are sensitive to changes in all the parameters except τ in the case of T* and TVC(T)*..

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