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HEAT ABSORPTION EFFECT ON MAGNETOHYDRODYNAMIC (MHD) FLOW OF JEFFERY FLUID IN AN INFINITE VERTICAL PLATE

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ABSTRACT

The current research reveals the impact of heat absorption on unsteady MHD convective Jeffery flow of a viscous, electrically conducting and incompressible fluid is researched on. The equations governing the flow of the fluid are described as Partial Differential Equations (PDEs) and Finite Difference Method (FDM) is used to obtain numerical solutions. Numerical investigations were conducted to examine the effect of parameters in the flow of the fluid i.e. on the velocity, temperature and concentration with the aid of graphs. It is seen that, the momentum boundary layer increases as the values of heat absorption and Jeffery parameters are increased while the velocity of the fluid fall for higher values of the suction and chemical reaction parameter. Also, the temperature of the fluid rises as heat generation becomes significant and a reverse trend is seen when suction is increased while increase in heat absorption parameter causes an increase in the concentration of the fluid.

Keywords: Jeffery fluid, heat transfer, MHD, heat absorption, magnetic field

INTRODUCTION

Heat transfer induced by mixed and natural convection in a fluid porous saturated medium has been studied in a broad range of technical applications, including petroleum industries, plasma research, MHD power generators, geothermal systems, insulation of heat, drying technologies, catalytic reactors, food business, and the solar energy collectors. Heat source/sink has applications in issues involving dissociating fluids, chemical processes, and may affect the rate of particle deposition. In renewable energy and waste heat recovery, heat source/sink chillers are employed. Several absorption methods include a generator-to-air heat exchanger, a compression-to-air heat pump, and a discrete heating system (Ramzan, 2022). Noor *et al.* (2020) numerically studied magnetohydrodynamic (MHD) squeezing flow of Jeffrey fluid between two parallel plates in a porous medium with the presence of thermal radiation, heat generation/absorption and chemical reaction.

Mainly, fluids are partitioned as Newtonian and non-Newtonian fluids. Non-Newtonian fluids have numerous practical and industrial applications, and such fluids involve honey, blood, greases and oils. Polymer industries, textile, irrigation problems and biological systems incorporate flows of non-Newtonian fluids in porous medium encountering magnetic effects. Anwar *et al.* (2021) investigated unsteady MHD fluid flow incorporating thermal radiative heat flux and heat injection/suction. Omokhuale et al. (2016) researched on the effect of heat absorption on steady/unsteady MHD free convection flow.

It is perceived that a single differential equation is described in the models of Newtonian fluid flows, but in the case of non-Newtonian fluid models, it is not so easy to describe the flow of the model with one and only constitutive differential equation. Usually, the rheological properties of fluids are specified with the help of their hypothetical constitutive conditions. Moreover, it is seen that the Newtonian fluids fulfilled Newton's internal friction law that is 'shear stress is proportional to the viscosity of the fluid gradient' and non-Newtonian fluids dissatisfy the Newtonian law of internal friction. The leading flow equations of non-Newtonian fluids are more difficult than the Navier-Stokes equations (Khan *et al.*, 2017; Zheng and Zang *et al.*, 2016; Jamil, 2016). Generally, non – Newtonian fluids are classified into three different types namely: first is differential type, second is the integral type and third is rate type. In this research, we will consider the model of Jeffery fluid flow, and this sort of fluid flow model indicates the property of the ratio of relaxation and retardation time. It is verified that the non-Newtonian models of fluid flows, with as well as without the magnetic-hydrodynamic fluid, have countless uses among the different fields of life, for example, biological fluids management, dental amalgam, plasmas, alloys and metals that are liquid form, electromagnetic propulsion and blood (Bajwa *et al.*, 2022). As with a number of rheological models developed, the Jeffrey's model has proved quite successful. This simple, yet elegant rheological model was introduced originally to simulate earth crustal flow problems (Jeffreys, 1929). This model Bird *et al.* (1987) constitutes a viscoelastic fluid model which exhibits shear thinning characteristics, yield stress and high shear viscosity. The Jeffrey's fluid model degenerates to a Newtonian fluid at a very high wall shear stress i.e. when the wall stress is much greater than yield stress. This fluid model also approximates reasonably well the rheological behavior of other liquids including physiological suspensions, foams, geological materials, cosmetics, and syrups. Uwanta and Omokhuale (2014) and Uwanta *et al.* (2014) conducted research of Jeffery fluid flow numerically using FDM.

The effects of Hall and ion slip on the radiative magnetohydrodynamic (MHD) rotating flow of viscous incompressible electrically conducting Jeffrey fluid over an infinite vertical flat porous surface by the ramped wall velocity and temperature, and isothermal plate have been explored by Krishna (2021). Krishna (2022) extended and researched on chemical reaction, heat absorption and Newtonian heating on MHD free convective flow. Goud (2020) examined heat generation/absorption influence on steady stretched permeable surface on MHD flow of a micropolar fluid in the presence of variable suction/injection. The aim of this study is to research on the effect of heat absorption on unsteady convective Jeffery flow in an infinite vertical plate. The flow is governed by a modeled coupled nonlinear system of partial differential equations (PDEs) in dimensional form which are transformed into non-dimensional form using some non-dimensional variables. The numerical solutions of the resulting equations were gotten by employing FDM for the velocity, temperature and concentration of the fluid. Furthermore, we examined the significant quantities involved in the flow process by drawing their graphs in relation to the parameters of physical significance for various flow conditions.

Mathematical Formulation

We considered an unsteady two–dimensional flow of an electrically conducting incompressible viscous fluid past an infinite vertical porous plate moving with Jeffery fluid. As the plate is infinite in extent, the physical variables are functions of y^* and t^* where y^* is taken normal to the plate and the x^*-direction is taken along the plate in the vertical upward direction, where fluid suction or injection and magnetic field are imposed at the plate surface. The temperature and concentration of the fluid are raised to T_w^* and C_w^* respectively and are higher than the ambient temperature and that of fluid. In addition, the effect of chemical absorption and chemical reaction are taken into account. It is assumed that induced magnetic field is negligible and the heat generated are not neglected.

We further assumed that the Boussinesq and boundary-layer approximations hold, the basic equations which govern the problem are given by:

$$\frac{\partial v^*}{\partial y^*}=0 \tag{1}$$

$$\frac{\partial u^*}{\partial t^*}+v^*\frac{\partial u^*}{\partial y^*}=\frac{\nu}{1+\lambda_1}\frac{\partial^2 u^*}{\partial y^{*2}}+g\beta\left(T^*-T_\infty^*\right)+g\beta_1\left(C^*-C_\infty^*\right)-\frac{\sigma B_0^2}{\rho}u^*-\frac{\nu}{K_1}u^* \tag{2}$$

$$\frac{\partial T^*}{\partial t^*}+v^*\frac{\partial T^*}{\partial y^*}=\frac{k}{\rho Cp}\frac{\partial^2 T^*}{\partial y^{*2}}+\frac{Q}{\rho Cp}\left(T^*-T_\infty^*\right) \tag{3}$$

$$\frac{\partial C^*}{\partial t^*}+v^*\frac{\partial C^*}{\partial y^*}=D\frac{\partial^2 C^*}{\partial y^{*2}}-Rc\left(C^*-C_\infty^*\right)+Q_1\left(T^*-T_\infty^*\right) \tag{4}$$

with the following initial and boundary conditions:

$$\left.\begin{array}{l} t^*\le 0, u^*=0, T^*\to T_\infty^*, C^*\to C_\infty^* \;\; for\,all\; y^* \\ t^*>0, u^*=0, \dfrac{\partial T^*}{\partial y^*}=-\dfrac{q}{k}, \dfrac{\partial C^*}{\partial y^*}=-\dfrac{q_m}{k_m} \;\; at\; y^*=0 \\ \quad u^*\to 0, T^*\to T_\infty^*, C'\to C_\infty^* \;\; as\; y^*\to\infty \end{array}\right\} \tag{5}$$

where u^* and v^* are the velocity components in x^* and y^* directions respectively, T is the temperature, t is the time, g is the acceleration due to gravity, β is the thermal expansion coefficient, β_1 is the concentration expansion coefficient, υ is the kinematic viscosity, D is the chemical molecular diffusivity, C_p is heat capacity at constant pressure, B_0 is a constant magnetic field intensity, σ is the electrical conductivity of the fluid, k is the thermal conductivity, K_1 is the permeability, ρ is the density, λ_1 is the Jeffery fluid, Rc is the chemical reaction parameter, Q_1 is the heat absorption parameter, T_w is the wall temperature, T_∞^* is the free stream temperature, q is heat flux, q_m is mass flux, k_m is mass diffusivity, C_w is the species concentration at the plate surface, C_∞^* is the free stream concentration, Q is the heat generation coefficient.

$v_0>0$ is the suction parameter and $v_0<0$ is the injection parameter. Now, in order to obtain non-dimensional PDEs, we are introducing the following non-dimensional variables and constants.

$$\left.\begin{array}{l} u=\dfrac{u'}{u_0}, y=\dfrac{u_0 y'}{\nu}, t=\dfrac{t'u_0}{t_0}, \nu=\dfrac{\mu}{\rho}, \theta=\dfrac{\left(T'-T_\infty'\right)k_0u_0}{q\nu}, C=\dfrac{\left(C'-C_\infty'\right)k_mu_0}{q_m\nu} \\ \Pr=\dfrac{\mu Cp}{k_0}, M=\dfrac{\sigma B_0^2\nu}{\rho u_0^{\,2}}, K=\dfrac{\nu^2}{K^*u_0^{\,2}}, Gr=\dfrac{g\beta\nu^2q}{k_0u_0^{\,4}}, Gc=\dfrac{g\beta^*\nu^2q}{k_mu_0^{\,4}} \\ Sc=\dfrac{\nu}{D}, \phi=\dfrac{Q\nu}{\rho Cpu_0^{\,2}}, \eta=\dfrac{Q_1\nu}{u_0^{\,2}}, \delta=\dfrac{R_1\nu}{u_0}, \xi=\dfrac{v_0}{u_0} \end{array}\right\} \tag{6}$$

where u_0 and t_0 are reference velocity and time respectively. Using (1) and (6), equations (2) to (4) are transformed into the following:

$$\frac{\partial u}{\partial t}-\xi\frac{\partial u}{\partial y}=\frac{1}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2}+Gr\theta+GcC-Mu-Ku \tag{7}$$

$$\frac{\partial \theta}{\partial t}-\xi\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2}+\phi\theta \tag{8}$$

$$\frac{\partial C}{\partial t}-\xi\frac{\partial C}{\partial y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}-\delta C+\eta\theta \tag{9}$$

The corresponding boundary conditions are:

$$\left.\begin{array}{l} t\le 0, u=0, \theta=0, C=0 \; for\, all\; y \\ t>0, u=0, \dfrac{\partial\theta}{\partial y}=-1, \dfrac{\partial C}{\partial y}=-1 \; at\; y=0 \\ u\to 0, \theta\to 0, C\to 0\, as\; y\to\infty \end{array}\right\} \tag{10}$$

where Gr is the thermal Grashof number, Gc is the mass Grashof number, Sc is the Schmidt number, $\Pr$ is the Prandtl number, M is the magnetic parameter, K is the permeability parameter, ξ is the suction parameter, δ is chemical reaction parameter, ϕ is the heat absorption parameter and η is the chemical absorption parameter.

Equations (7) to (9) are now approximated by finite difference schemes of Crank – Nicolson type. The finite difference approximations of these equations are as follows:

$$\left(\frac{u_{i,j+1}-u_{i,j}}{\Delta t}-\gamma\frac{u_{i+1,j}-u_{i,j}}{\Delta y}\right)=\frac{1}{(1+\lambda_1)}\left[\frac{u_{i+1,j}+u_{i-1,j}-2u_{i,j}+u_{i+1,j+1}+u_{i-1,j+1}-2u_{i,j+1}}{2(\Delta y)^2}\right]$$
$$+\frac{Gr}{2}\left(\theta_{i,j+1}+\theta_{i,j}\right)+\frac{Gc}{2}\left(C_{i,j+1}+C_{i,j}\right)-\frac{M}{2}\left(u_{i,j+1}+u_{i,j}\right)-\frac{K}{2}\left(u_{i,j+1}+u_{i,j}\right) \tag{11}$$

$$\left(\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta t}-\gamma\frac{\theta_{i+1,j}-\theta_{i,j}}{\Delta y}\right)=\frac{1}{\Pr}\left[\frac{\theta_{i+1,j}-\theta_{i-1,j}-2\theta_{i,j}+\theta_{i+1,j+1}+\theta_{i-1,j+1}-2\theta_{i,j+1}}{2(\Delta y)^2}\right]$$
$$+\frac{\phi}{2}\left(\theta_{i,j+1}+\theta_{i,j}\right) \tag{12}$$

$$\left(\frac{C_{i,j+1}-C_{i,j}}{\Delta t}-\gamma\frac{C_{i+1,j}-C_{i,j}}{\Delta y}\right)=\frac{1}{Sc}\left[\frac{C_{i+1,j}-C_{i-1,j}-2C_{i,j}+C_{i+1,j+1}+C_{i-1,j+1}-2C_{i,j+1}}{2(\Delta y)^2}\right]$$
$$-\frac{\delta}{2}\left(C_{i,j+1}+C_{i,j}\right)+\frac{\eta}{2}\left(\theta_{i,j+1}+\theta_{i,j}\right) \tag{13}$$

The initial and boundary conditions become

$$\begin{array}{l} u_{i,0}=0, \theta=0, C_{i,0}=0 \text{ for all i except } i=0 \\ u_{i,0}=0, \dfrac{\theta_{i+1,j}-\theta_{i,j}}{\Delta y}=-1, \dfrac{C_{i+1,j}-C_{i,j}}{\Delta y}=-1 \\ u_{l,0}=0, \theta_{l,0}=0, C_{l,0}=0 \end{array} \tag{14}$$

where l corresponds to ∞ . The suffix i corresponds to y and j is equals to t. consequently, $\Delta t=t_{j+1}-t_j$ and $\Delta y=y_{i+1}-y_i$.

In order to access the effects of parameters on the flow variables namely; Jeffery parameter, thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number, magnetic parameter, heat absorption, permeability parameter, suction parameter, chemical absorption parameter and chemical reaction parameter on the velocity, temperature and concentration, and have grips of the physical problem, the unsteady coupled non-linear partial differential equations (11) – (13) with boundary conditions (14) have been solved using Maple package with the following values for Gr = Gc = M = η = 1, $\lambda_1=0.5$, Pr = 0.71, Sc = 0.78, δ = 0.1, δ = 0.5, K= 0.5, $\xi=0.5$, $\eta=0.5$ except where they are varied. A step size of $\Delta t=0.01$, Δy = 0.25 is used for the interval $y_{\min}$ = 0 to $y_{\max}$ = 5 for a desired accuracy and a convergence criterion of 10^{-6} is satisfied for various parameters.

RESULTS AND DISCUSSION

For the values of C, θ, u at time t, the values at a time $t+\Delta t$ gotten for $i=1,2,...,l-1$ in (14) which results in a tri-diagonal system of equations in unknown values of C. Similarly, calculating θ and u from (13) and (12) respectively. Furthermore, to validate this study in the absence of Eckert number and variable thermal conductivity in the heat equations in the work of Uwanta and Omokhuale (2014) and when $\eta=0,\ \frac{\partial\theta}{\partial y}=-1$ and $\frac{\partial C}{\partial y}=-1$ are modified in the boundary conditions of this research the results of Uwanta and Omokhuale (2014) are gotten. Also, in the absence of Soret number, Eckert number and variable thermal conductivity in the heat equations in the work of Uwanta *et al.* (2014) and when $\eta=0,\ \frac{\partial\theta}{\partial y}=-1$ and $\frac{\partial C}{\partial y}=-1$ are modified in the boundary conditions of this research the results of Uwanta *et al.* (2014) are obtained.

Velocity profiles

Figures 1 to 8 represent the velocity profiles with varying parameters respectively. Figure 1 depicts the effect of Prandtl number on the velocity. It is observed that, the velocity decreases with increasing Prandtl number. The Prandtl number correlates with momentum to the thermal diffusivities; hence, higher values of Pr lead to lower thermal diffusivity, which causes decay in the temperature field. Effect of Schmidt number on the velocity is presented in Figure 2, It is found that, the velocity falls with the increase in Schmidt number. Figure 3 shows variation of chemical absorption parameter on the velocity profile. It is noted that, the velocity increases with increase in chemical absorption parameter. Effect of thermal Grashof number on the velocity profile is depicted in Figure 4, It is observed that, the velocity increases with increasing thermal Grashof number. Figure 5 illustrates different values of mass Grashof number on the velocity. It is found that, the velocity increases with the increase of the mass Grashof number. Effect of Jeffery parameter on the velocity is presented in Figure 6. It is clear that, the velocity increases with increase in Jeffery parameter. Figures 7 and 8 show the effects of suction and heat absorption parameters on the velocity of the fluid. It is clear that, the velocity becomes lower as the suction parameter is increased while a reverse trend is observed as heat absorption becomes significant.

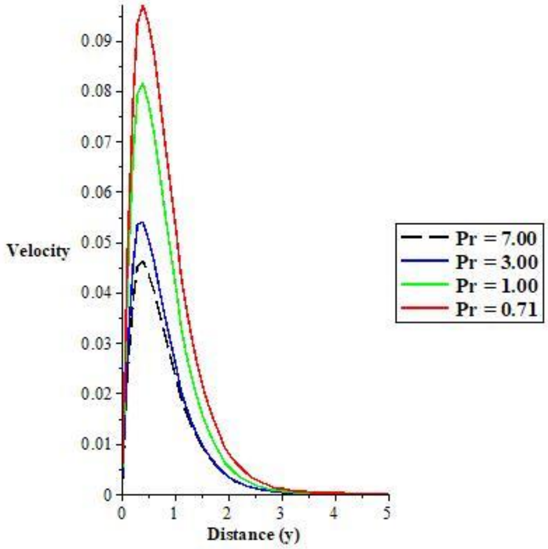

Figure 1: Velocity profiles for different values of Pr.

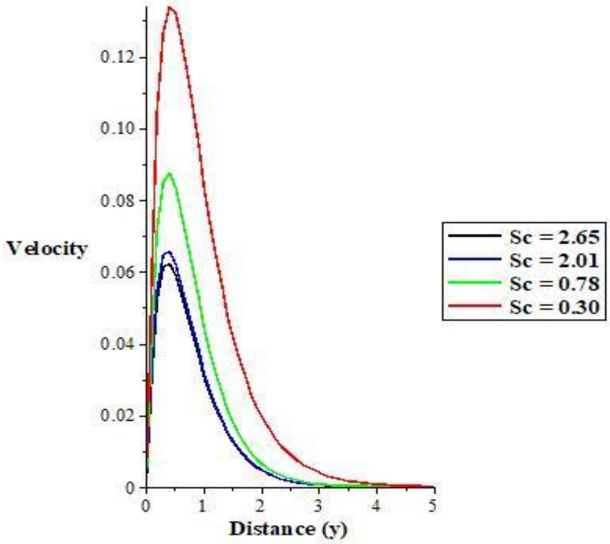

Figure 2: Velocity profiles for different values of Sc.

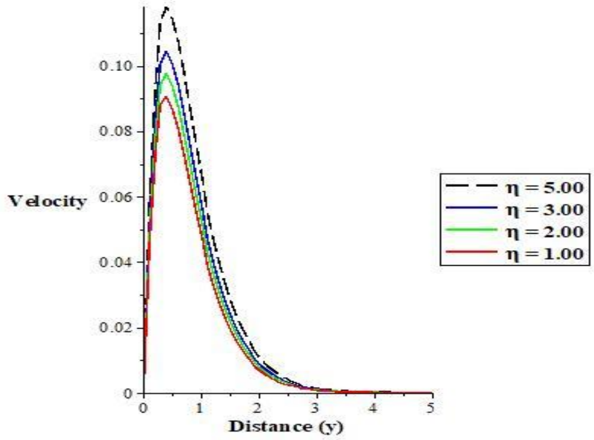

Figure 3: Velocity profiles for different values of η.

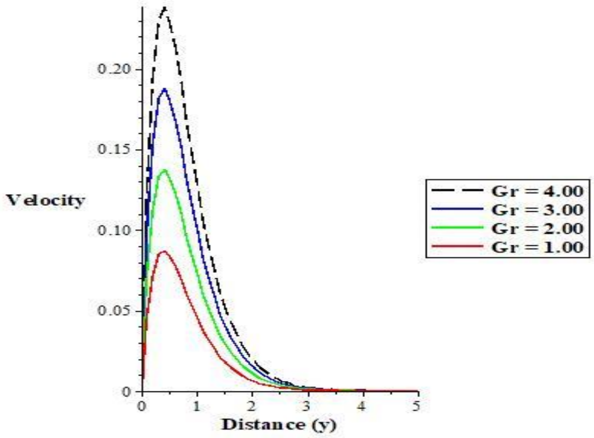

Figure 4: Velocity profiles for different values of Gr.

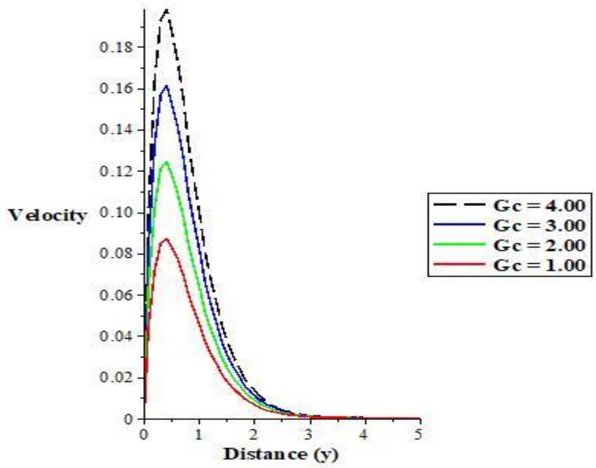

Figure 5: Velocity profiles for different values of Gc.

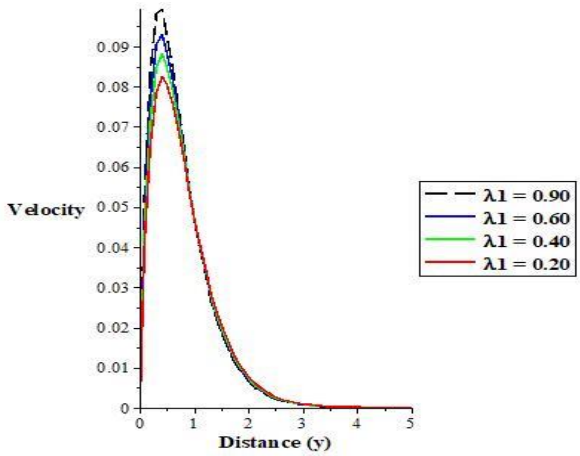

Figure 6: Velocity profiles for different values of λ_1

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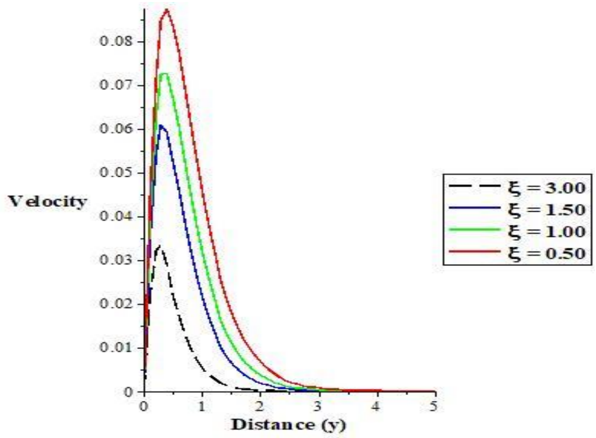

Figure 7: Velocity profiles for different values of ξ .

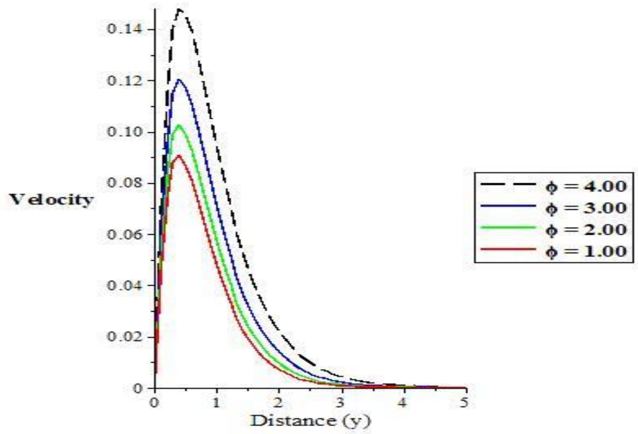

Figure 8: Velocity profiles for different values of ϕ. .

Temperature profiles

Figures 9 and 10 illustrate the temperature profiles. In Figure 9, the effect of suction parameter on the temperature is demonstrated. It is seen that, the temperature decreases when the suction parameter is increased. Figure 10 represents effect of heat absorption on the temperature. It is depicted that, the temperature increases with increase in heat absorption parameter.

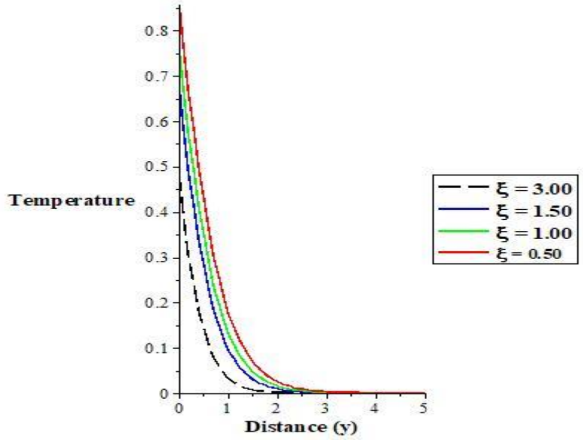

Figure 9: Temperature profiles for different values of ξ.

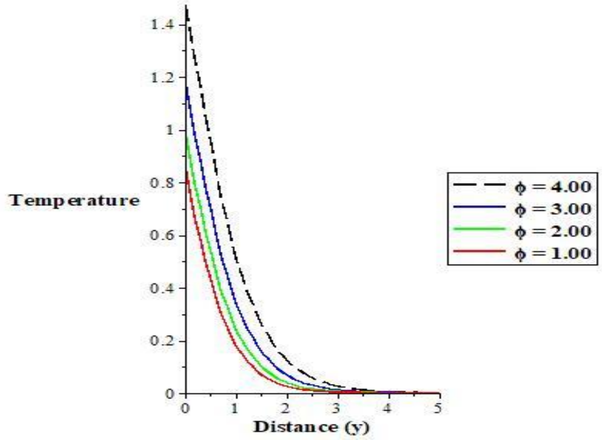

Figure 10: Temperature profiles for different values of ϕ

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Concentration profiles

Figures 11 to 12 show the concentration profiles. Effect of suction on the concentration is presented in Figure 11. It is noted that, the concentration is lower due to increasing suction. In Figure 12, the effect of chemical absorption parameter on the concentration is shown. It is demonstrated that. The concentration is higher as the chemical absorption parameter is increased.

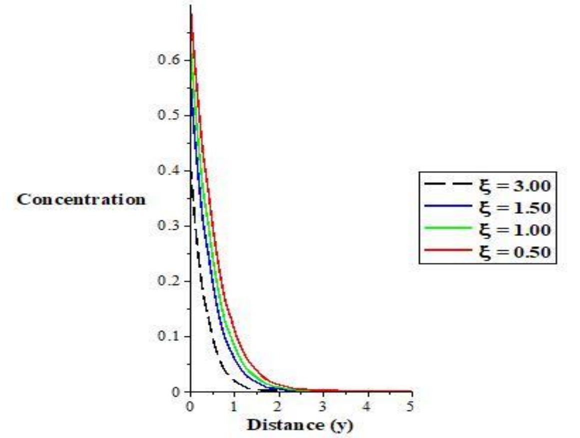

Figure 11: Concentration profiles for different values of ξ.

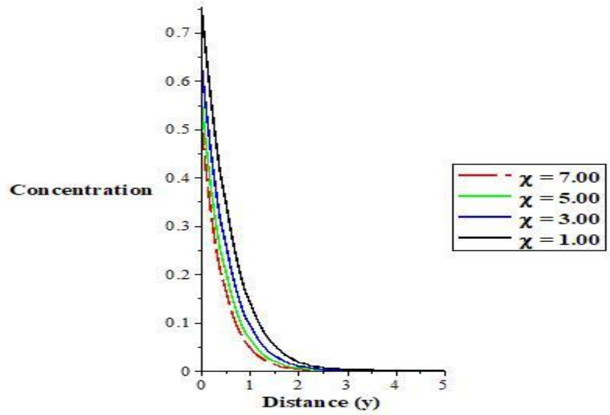

Figure 12: Concentration profiles for different values of η.

CONCLUSION

We studied unsteady MHD convective Jeffery flow with heat absorption effect. The problem is modeled by a system coupled non-linear PDEs and solved by FDM. Numerical experiments were carried out to examine the effects of physical parameters on the fluid velocity, temperature and concentration. The following conclusion can be deduced from this study: The fluid concentration reduces due to higher values of heat absorption parameter and becomes lower for increased values of suction. Higher values of heat generation parameter led to increase the temperature of the fluid while an opposite trend is found as the suction is increased. The momentum boundary layer rises as the values of heat absorption parameter, Jeffery parameter, chemical absorption parameter, thermal and mass Grashof numbers are increased while the velocity of the fluid falls as suction, Schmidt and Prandtl numbers become significant. When the numerical results of this study are compared with results in literatures a good agreement was observed.

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