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MONTE CARLO APPROACH FOR COMPARATIVE ANALYSIS OF REGRESSION TECHNIQUES IN THE PRESENCE OF MULTICOLLINEARITY AND AUTOCORRELATION PHENOMENA

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ABSTRACT

Multicollinearity and Autocorrelation are two very common problems in regression analysis. As its well-known, the presence of some degrees of multicollinearity results in estimation instability and model mis-specification while the presence of serial correlated errors lead to underestimation of the variance of parameter estimates and inefficient prediction. These two conditions have adverse effects on estimation and prediction; therefore, a wide range of tests have been developed to reduce their impact. Invariably, the multicollinearity and autocorrelation problems are dealt with separately in most studies. Thus, this study explored the predictive ability of the proposed GLS-Ridge regression on multicollinearity and autocorrelation problems simultaneously, using simulated dataset. Data used for the study was the data simulated using Monte Carlo. In the application, 1000 repetitions have been simulated for each of the sample size of n=40. The model (GLS-Ridge), $\hat{Y}=\hat{\beta}_{RGLS_0}+\hat{\beta}_{RGLS_1}X_1+\hat{\beta}_{RGLS_2}X_2+\hat{\beta}_{RGLS_3}X_3+\cdots+\hat{\beta}_{RGLS_{p-1}}X_{p-1}$ was proposed, and an estimator, $\hat{\beta}_{RGLS}=(X'\Omega^{-1}X+kI)^{-1}X'\Omega^{-1}Y$ was derived. Least squares, ridge, lasso and the GLS-R model were applied to the simulated dataset. Regression coefficients for each estimator were computed and statistical comparison criteria; Mean Square Error and Akaike Information Criteria of the estimates were used to select the best model. For the simulated data, the GLS-R model had smaller AIC value than least squares, ridge regression and LASSO techniques for samples n=40. Among these four techniques, the GLS-R model gives the smallest AIC value. The research work revealed that the GLS-R regression technique has a better predictive ability in the presence of autocorrelation and multicollinearity, hence it is preferred than the other three techniques.

Keywords: Ridge regression, lasso, Monte Carlo simulation, multicollinearity, autocorrelation

INTRODUCTION

Multicollinearity and Autocorrelation are two very common problems in regression analysis. The term multicollinearity is used to denote the presence of linear relationships among explanatory variables (Chatterjee and Hadi, 2006). This is a clear violation of the Ordinary Least Squares (OLS) assumption that state that the explanatory variables are not perfectly linearly correlated (Maddala, 2002). In the view of Agunbiade (2012), he emphasized that multicollinearity is matter of degree that is inherent in any dataset. Multicollinearity may arise for various reasons. Firstly, there is the tendency of economic variables to move along over a period of time. Phenomenon like this are affected by the same factors and, consequently, these factors becomes obvious in the variables and shows the same broad pattern of behaviour over time. For example, during the booms period, the basic economic magnitudes grow even though they tend to lag behind others. Thus, income, saving, consumption, prices, investment, employment, tends to grow (or increase) in the time of economic expansion and decrease or reduces in the period of recession. Growth and trend factors in time series are the most serious cause of multicollinearity. Secondly, the use of lagged values of some explanatory variables as separate independent factors in the relationship. For example, in investment functions, distributed lags concerning past levels of economic activity are introduced as separate explanatory variables. Also, in consumption function it has become necessary to include among the explanatory variables past level as well as the present levels of income. Naturally the successive values of a certain variable are intercorrelated. For instance, income in the current period is partly determined by its own value in the previous period and so on. With strongly interrelated regressors, the regression coefficients provided by the OLS estimator are no longer stable even when they are still unbiased as long as multicollinearity is not perfect. Furthermore, the regression coefficient may have large sampling errors which affect both the inference and forecasting that is based on the model (Chartterjee *et al*., 2000). Various other estimation methods have been developed to tackle this problem. This estimation include Ridge regression estimator developed by Hoerl (1962), Hoerl and Kennard (1970), estimator based on principal component regression suggested by Massy (1965), Marquardt (1970) and Bock *et al*. (1973), Naes and Marten (1988) and method of partial least squares developed by Hermon Wold in the 1960's, Holland (1990), Phatak and Jony (1997), Ayinde (2017) and Alhassan *et al.* (2019).

The assumption of Ordinary Least Squares is that the successive values of the random variables 'u' are temporally independent, that is, the value which it assumes in any previous periods. This assumption implies that the covariance of u_i and $u_j$is equal to zero. If this assumption is not satisfied, that is, if the value of the random variable u in any particular period is correlated with its own preceding value (or values), then there is Autocorrelation or Serial correlation of the random variables (Gujarati, 2003).

Autocorrelation is when the error terms or random variables u_i and u_j are correlated. Several authors have worked on this problem especially in term of the parameter estimation of the linear regression model when the error term follows autoregressive of order one. The OLS estimator is inefficient even though unbiased. Its predicted value is also inefficient and the sampling variances of the autocorrelated term are known to be underestimated causing the t and F tests to be invalid.

In a linear regression model, there are situations where the regressors may be correlated and the error terms may be autocorrelated. This phenomenon is known as autocorrelated model with multicollinearity; which is a rarely discussed area in regression analysis. It is well known that when there is multicollinearity, the OLS estimators for regression coefficients or the predictors based on these estimates may give poor results. For overcoming the problem of multicollinearity, several methods are available such as Lasso, Ridge regression and Partial least squares. These methods are useful when the errors are non autocorrelated. However, in the presence of autocorrelated model with multicollinearity, appropriate modifications need to be incorporated in its estimation. Premise on gap, this study intends to propose a regression called Generalized Least Squares-Ridge with the predictive ability of the estimator in the presence of multicollinearity and autocorrelation simultaneously.

The aim of the study is to explore the predictive ability of the proposed GLS-Ridge regression on multicollinearity and autocorrelation problems simultaneously, using real life and simulated dataset; and compare this with the existing regression known techniques. The specific objectives are to: develop a proposed model for solving the problems of mutlicollinearity and autocorrelation simultaneously, carry out a Monte Carlo simulation based on the assumption being violated on multicollinearity and autocorrelation, estimate the parameters of the simulated data and compare the predictive ability of regression methods [linear regression, the least absolute shrinkage and selection operator (LASSO), ridge regression and proposed generalized-ridge regression method] with varying degrees of multicollinearity and autocorrelation at sample $n = 40$.

MONTE CARLO STUDIES

The concepts, importance, usage and application of Monte Carlo Methods in various fields are well documented in the literature. Its use in regression analysis as a supplement to shortage in real life data availability is also of great concern.

Various studies by Agunbiade (2007, 2011, and 2012), Ayinde (2007) have also provided detained steps needed in using MCM to simulate data for regression analysis. Such justification have also been provided in Maddala (2002) and many more. Its application in the field of Information Technology was noticed by Alexandrov *et al*. (2011) which described, various approaches of designing scalable algorithms. The work proposes implementations of parallel Monte Carlo algorithms and demonstrated their huge potential regarding speedup, fault-tolerance and scalability on a variety of applications. It also adds future research possibilities, for example, investigate next generation algorithms for resilience and fault-tolerance in large-scale systems. The set of problems in computational finance will be expanded in order to generalise the approach. With ever increasing numbers of processors and machines, traditional ways of treating faults are not viable any more, as they impose too many constraints and too much overhead when employed in larger systems. Furthermore, additional fault tolerance techniques will be examined in response to deterministic and nondeterministic failure occurrence.

Effect of Multicollinearity and Autocorrelation on Regression Techniques for Predictive Ability

Agunbiade (2012) carried out a research on 'a note on the effects of the multicollinearity phenomenon of a simultaneous equation model. The study sought to examine both the intra and inter equation effects of multicollinearity on the exogenous variables of a simultaneous equation model and also investigate the effect of the unsuspected dependence between random normal deviates used for generating the stochastic component of a simultaneous equation model. A Monte Carlo approach was used to achieve this by setting up a two – equation with five structural parameters simultaneous econometric model and applying the six different estimation techniques. It was found that while 2SLS and LIML produced identical results, the results of other techniques were at variants, which confirmed the likely presence of multicollinearity among the regressor variables.

Agunbiade and Iyaniwura (2010) worked on estimation under multicollinearity. A comparative investigation was done experimentally for 6 different estimation Techniques of a just – identified simultaneous three – equation econometric model with three multi-collinearity exogenous variables. The aim is to explore in depth the effects of the problems of multicollieearity and examine the sensitivity of findings to increasing sample sizes and increasing number of replications using the mean and total absolute bias statistics. Their findings revealed that the estimates are virtually identical for three estimators: LIML, 2SLS, and ILS, while the performance of the other categories are not uniformly affected by the three levels of multicollinearity considered. Their study had established that L2ILS estimators are best for estimating parameters of data plagued by the lower open interval negative levels of multicollinearity while FIML and OLS respectively rank highest for estimating parameters of data characterized by close interval and upper categories level of multicollinearity.

Alhassan *et al.* (2019) investigated the effects and consequences of multicollinearity on both standard error and explanatory variables in multiple regression, the correlation between X1 to X6 (independent variables) measure their individual effect and performance on Y (Response variable) and it is carefully observes how those explanatory variables intercorrelated with one another and to the response variable. There are many procedures available in literature for detecting presence, degree and severity of multicollinearity in multiple regression analysis. They used correlation analysis to discover it is presence; they use variance inflation factors, tolerance level, indices number, eigenvalues to access fluctuation and influence of multicollinearity present in the model.

Ayinde *et al*. (2012) examined effect of multicolinearity and autocorrelation on predictive ability of some estimators of linear regression model. Violation of the assumptions of independent regressors and error terms in linear regression model has respectively resulted into the problems of multicollinearity and autocorrelation. Each of these problems separately has significant effect on parameters estimation of the model parameters and hence prediction. Their study therefore attempts to investigate the joint effect of the existence of multicollinerity and autocorrlation on Ordinary Least Square (OLS) estimator, Cochrane-Orcutt (COR) estimator, Maximum Likelihood (ML) estimator and the estimators based on Principal Component (PC) analysis on prediction of linear regression model through Monte Carlo studies using the adjusted coefficient of determination goodness of fit statistic of each estimator. With correlated normal variables as regressors, it further identifies the best estimator for prediction at various levels of sample sizes (n), multicollinearity (λ) and autocorrlation (ρ). Results reveal the pattern of performances of COR and ML at each level of multicollinearity over the levels of autocorrelation to be generally and evidently convex especially when $n \geq 30$ and $\lambda < 0$ while that of OLS and PC is generally concave. Moreover, the COR and ML estimators perform equivalently and better; and their performances become much better as multicollinearity increases. The COR estimator is generally the best estimator for prediction except at high level of multicollinearity and low levels of autocorrelation. At these instances, the PC estimator is either best or competes with the COR estimator. Moreover, when the sample size is small ($n = 10$) and multicollinearity level is not high, the OLS estimator is best at low level of autocorrelation whereas the ML is best at moderate levels of autocorrelation.

Ayinde *et al*. (2012) examined the performances of the Ordinary Least Square (OLS) estimator, Cochrane-Orcutt (COR) estimator, Maximum Likelihood (ML) estimator and the estimators based on Principal Component (PC) analysis in prediction of linear regression model under the joint violations of the assumption of non-stochastic regressors, independent regressors and error terms. With correlated stochastic normal variables as regressors and autocorrelated error terms, Monte-Carlo experiments were conducted and the study further identifies the best estimator that can be used for prediction purpose by adopting the goodness of fit statistics of the estimators. From the results, it is observed that the performances of COR at each level of correlation (multicollinearity) and that of ML, especially when the sample size is large, over the levels of autocorrelation have a convex-like pattern while that of OLS and PC are concave-like. Also, as the levels of multicollinearity increase, the estimators, except the PC estimators when multicollinearity is negative, rapidly perform better over the levels autocorrelation. The COR and ML estimators are generally best for prediction in the presence of multicollinearity and autocorrelated error terms. However, at low levels of autocorrelation, the OLS estimator is either best or competes consistently with the best estimator, while the PC estimator is either best or competes with the best when multicollinearity level is high ($\boldsymbol{k} \geq 0.8$ or $k \leq -0.49$).

MATERIALS AND METHODS

Monte Carlo Simulation

Simulations studies are usually used to investigate the properties and behaviour of various statistics of interest. The technique is often used in econometrics when the properties of a particular estimation method are not known. For example, it may be known from asymptotic theory how a particular test behaves with an infinite sample size, but how will the test behave if only 50 observations are available? Will the test still have the desirable properties of being correctly sized and having high power? In other words, if the null hypothesis is correct, will the test lead to rejection of the null 5% of the time if a 5% rejection region is used? And if the null is incorrect, will it be rejected a high proportion of the time?

Examples from econometrics of where simulation may be useful include:

- Quantifying the simultaneous equations bias induced by treating an endogenous variable as exogenous
- Determining the appropriate critical values for a Dickey--Fuller test
- Determining what effect heteroscedasticity has upon the size and power of a test for autocorrelation.

Conducting a Monte Carlo simulation

(1) Generate the data according to the desired data generating process (DGP), with the errors being drawn from some given distribution
(2) Do the regression and calculate the test statistic.

(3) Save the test statistic or whatever parameter is of interest
(4) Go back to stage 1 and repeat *N* times.

Simulations are also often extremely useful tools in finance, in situations such as:

● The pricing of exotic options, where an analytical pricing formula is unavailable
● Determining the effect on financial markets of substantial changes in the macroeconomic environment
● 'Stress-testing' risk management models to determine whether they generate capital requirements sufficient to cover losses in all situations.

In all of these instances, the basic way that such a study would be conducted (with additional steps and modifications where necessary) is shown above.

A brief explanation of each of these steps is in order. The first stage involves specifying the model that will be used to generate the data. This may be a pure time series model or a structural model. Pure time series models are usually simpler to implement, as a full structural model would also require the researcher to specify a data generating process for the explanatory variables as well. Assuming that a time series model is deemed appropriate, the next choice to be made is of the probability distribution specified for the errors. Usually, standard normal draws are used, although any other empirically plausible distribution (such as a Student's t) could also be used. The second stage involves estimation of the parameter of interest in the study. The parameter of interest might be, for example, the value of a coefficient in a regression, or the value of an option at its expiry date. It could instead be the value of a portfolio under a particular set of scenarios governing the way that the prices of the component assets move over time.

The quantity N is known as the number of replications, and this should be as large as is feasible. The central idea behind Monte Carlo is that of random sampling from a given distribution. Therefore, if the number of replications is set too small, the results will be sensitive to 'odd' combinations of random number draws. It is also worth noting that asymptotic arguments apply in Monte Carlo studies as well as in other areas of econometrics. That is, the results of simulation study will be equal to their analytical counterparts (assuming that the latter exist) asymptotically.

Generalised Least Squares-Ridge Regression Model

The proposed regression model can be written as:

$$\hat{Y} \circledast = \hat{\beta}_{RGLS_0} + \hat{\beta}_{RGLS_1}X_1 + \hat{\beta}_{RGLS_2}X_2 + \hat{\beta}_{RGLS_3}X_3 + \cdots + \hat{\beta}_{RGLS\,p-1}X_{p-1} \quad (1)$$

where,

$\hat{\beta}_{RGLS_{p-1}}$: Ridge-GLS regression parameter estimate,

X_{p-1}: Standardized predictor variable.

Consider a general linear regression model with errors and the regressors exhibiting near multicollinearity. As seen earlier, in case of autocorrelation. Hence autocorrelation is a particular case of heteroscedasticity. In the case of heteroscedasticity, GLS is an appropriate method of estimation as given in

$$\hat{\beta}_{GLS} = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}Y. \quad (2)$$

Further, when there is multicollinearity, often used method is the ridge regression.

$$\hat{\beta}_k^{ridge} = \left(X^TX + kI_p\right)^{-1}X^Ty$$

Combining these two methods, we propose for the autocorrelated model with multicollinearity a generalized ridge type estimator represented as

$$\hat{\beta}_{RGLS} = (X'\Omega^{-1}X + kI)^{-1}X'\Omega^{-1}Y. \quad (3)$$

Hence the model under consideration contains the unknown parameters k,ρ, σ^2 and β.

From Al-Hassan (2010) the following are some existing methods for estimating ridge parameter k.

For the proposed estimator, k_i's are the eigen values of $(X'\Omega^{-1}Y)^{-1}$.

RESULTS AND DISCUSSION

Data Simulations

In this chapter, first using R program, a population was generated. Second, random samples were drawn from this population with sample sizes of $n = 40$, $n = 80$ and $n = 120$. Least squares, ridge, lasso regression and the proposed regression were applied to each of these samples. Regression coefficients for each of these estimators are computed and as statistical criteria to compare these estimators, the mean and the standard deviation of the estimates were used. For these purposes the following steps were followed.

1. The population is generated.
2. A sample size $n = 40$ is selected from the population.
3. The least squares, ridge and lasso regression methods are applied to sampled data.
4. Returning to step 2, this process is repeated for 1000 times.

The simulation values obtained for the least squares, ridge and principal components regression are compared and researched shows, which methods give the best consequence by simulation.

Generating the Population

An R program was written to draw a random sample from the created population.

There are three predictor variables $X_1 X_2$ and X_3. X_1 has 10 different values as $10, 20, 30, \ldots, 100$. X_2 takes 4 different values according to each X_1 value. X_3 takes 5 different values according to each X_1 value Therefore there are 200 different groups of given X values. For each of X values there are 25 Y values and these response values are distributed normally and independently with mean $E(Y)$ and constant variance $\sigma = 25$. The correlation matrix of Y, X_1, X_2 and X_3 is given in Table 1. It can be seen that there exists a high correlation between X_1 and X_3.

Table 1 Correlation Coefficients between Variables (Simulated Data)

	X_1	X_2	X_3	Y
X_1	1.0000	-0.1708	0.9962	0.0472
X_2	-0.1708	1.0000	-0.1682	0.2831
X_3	0.9962	-0.1682	1.0000	0.0432
Y	0.0472	0.2831	0.0432	1.0000

Simulation results and Interpretation for Sample Size 40
1000 samples with $n = 40$ are selected from the constructed population. The mean and the standard deviation of estimated regression coefficients are given in the following Table
Means of these coefficients are calculated as $\bar{\beta}_0 = 0.4194008$, $\overline{\beta_1} = 2.045913$,$\bar{\beta}_2 = 1.004132$ and $\bar{\beta}_3 = -0.1752255$. Standard error of coefficients are $S_{\hat{\beta}_1} = 5.500221$, $S_{\hat{\beta}_2} = 0.5375916$ and $S_{\hat{\beta}_3} = 0.5483619$. The MSE is obtained as 1.769454. The AIC value obtained is 0.74937.
The results of ridge regression are given in Table 2. In this table for different k values, R^2, S and VIF values are given.

Table 2: Least Squares Regression Coefficients for $n = 40$

	$\bar{\beta}_i$	S_{β_i}
$Intercept$	0.4194008	0.4916419
X_1	2.045913	5.500221
X_2	1.004132	0.5375916
X_3	-0.1752255	0.5483619
$R^2 = 0.0921 S = 0.8896$		
$MSE = 1.769454$		
AIC = 0.74937		

Table 3: Summary Statistics for the Ridge Regression when $n = 40$

K	R2	Sigma	B'B	Ave VIF	Max VIF
0.000000	0.0921	0.8896	0.8995	89.2560	133.4292
0.001000	0.0914	0.8900	0.5963	55.8624	83.3172
0.002000	0.0909	0.8902	0.4368	38.3013	56.9647
0.003000	0.0905	0.8904	0.3426	27.9422	41.4200
0.004000	0.0902	0.8906	0.2823	21.3243	31.4897
0.005000	0.0899	0.8907	0.2415	16.8411	24.7629
0.005000	0.0899	0.8907	0.2415	16.8411	24.7629
0.006000	0.0897	0.8908	0.2124	13.6641	19.9963
0.007000	0.0895	0.8909	0.1911	11.3310	16.4961
0.008000	0.0893	0.8910	0.1749	9.5673	13.8505
0.009000	0.0892	0.8911	0.1623	8.2017	11.8022
0.010000	0.0890	0.8912	0.1523	7.1228	10.1841
0.020000	0.0877	0.8918	0.1105	2.7162	3.5814
0.030000	0.0867	0.8923	0.0985	1.5879	1.8983
0.040000	0.0858	0.8927	0.0927	1.1349	1.2281
0.050000	0.0849	0.8932	0.0890	0.9062	0.9300
0.060000	0.0840	0.8936	0.0861	0.7732	0.9118
0.070000	0.0831	0.8940	0.0838	0.6878	0.8942
0.080000	0.0823	0.8944	0.0818	0.6288	0.8771
0.090000	0.0815	0.8948	0.0799	0.5856	0.8604
0.100000	0.0807	0.8952	0.0782	0.5524	0.8443
0.200000	0.0736	0.8987	0.0645	0.4053	0.7050
0.300000	0.0677	0.9015	0.0544	0.3405	0.5978
0.400000	0.0626	0.9040	0.0466	0.2956	0.5134
0.500000	0.0583	0.9060	0.0404	0.2608	0.4457
0.600000	0.0545	0.9079	0.0353	0.2327	0.3907
0.700000	0.0512	0.9095	0.0311	0.2093	0.3453
0.800000	0.0483	0.9109	0.0277	0.1895	0.3075
0.900000	0.0457	0.9121	0.0248	0.1726	0.2755
1.000000	0.0433	0.9132	0.0223	0.1580	0.2483

$k = 0.005000$		
Independent Variable	**Regression Coefficient**	**Standard Error**
Intercept	0.3783011	
X1	0.9585001	2.372355
X2	0.9956309	0.5352418
X3	-0.06696446	0.2365231
	MSE	1.04804
	AIC	0.66996

Regarding all of these various statistics, from this table it is seen that the optimal k value is 0.005 for n=40. The values of R^2, S and VIFs are 0.0899, 0.8907 and 24.7629 when $k = 0.005$. The means of these coefficients are calculated as $\bar{\beta}_0 = 0.3783011$, $\overline{\beta_1} = 0.9585001$, $\bar{\beta}_2 = 0.9956309$ and $\bar{\beta}_3 = -0.06696446$. Standard error of coefficients are $S_{\widehat{\beta}_1} = 2.372355$, $S_{\widehat{\beta}_2} = 0.5352418$and $S_{\widehat{\beta}_3} = 0.2365231$. The MSE is obtained as 1.04804. The AIC value obtained is 0.66996.

Table 4 Summary Statistics for Lasso regression when $n = 40$

No.	Eigenvalue	Incremental Percent	Cumulative Percent	Condition Number
1	2.050915	68.36	68.36	1.00
2	0.945329	31.51	99.87	2.17
3	0.003756	0.13	100.00	546.00
Principal Component	**PC Coefficient**	**Individual R-Squared**	**Eigenvalue**	
PC1	-0.0003	0.0000	2.050915	
PC2	-0.2754	0.0891	0.945329	
PC3	-0.8050	0.0030	0.003756	
Independent Variable	**Regression Coefficient**	**Standard Error**	**VIF**	
Intercept	0.3411272			
X1	0.1429002	0.242048	0.2575	
X2	0.9984433	0.538235	1.0296	
X3	0.01450066	0.02417576	0.2582	
		MSE	0.2681	
		AIC	0.18749	

The first eigenvalue is 2.050915 and the first component accounts for 68.36% of the total variation in Y. The values of CN, R^2, S and VIFs are 1.00000, 0.0000, 0.242048, and 0.2575 when the first component is chosen. Considering these statistics principal components with 1 component omitted results the mean value of coefficients are $\bar{\beta}_0 = 0.3411272\overline{\beta_1} = 0.1429002, \bar{\beta}_2 = 0.9984433$ and $\bar{\beta}_3 = 0.01450066$. Standard deviations of coefficients are $S_{\hat{\beta}_1} = 0.242048$, $S_{\hat{\beta}_2} = 0.538235$and $S_{\hat{\beta}_3} = 0.02417576$. The MSE is obtained as 0.268153. The AIC value obtained is 0.18749.

The results of least squares, ridge and lasso coefficients and standard deviations are summarized in Table 5.

Table 5 Comparison of Least Squares, Ridge and Lasso Regression Coefficients when $n = 40$

	Least Squares		Ridge Regression		Lasso Regression	
	$\bar{\beta}_i$	S_{β_i}	$\bar{\beta}_i$	S_{β_i}	$\bar{\beta}_i$	S_{β_i}
Intercept	0.4194008	0.4916419	0.3783011		0.3411272	
X1	2.045913	5.500221	0.9585001	2.372355	0.1429002	0.242048
X2	1.004132	0.5375916	0.9956309	0.5352418	0.9984433	0.538235
X3	-0.1752255	0.5483619	-0.06696446	0.2365231	0.0145006	0.024175
MSE	1.769454		1.04804		0.268153	
AIC	0.74937		0.66996		0.18749	

As described in the literature, mean square error is smaller than the least squares method in both ways of biased regression. According to the regression coefficients, the ridge regression and the lasso regression result are similar to each other. As supposed, ridge regression method has result in smaller AIC values than the least squares method. Among these three methods, Lasso gives the smallest AIC value.

Table 6: Proposed model when $n = 40$

	$n = 40$	
i	$\bar{\beta}_i$	S_{β_i}
Intercept	0.3681857	0.5588043
X1	0.9348769	0.231596
X2	0.9556473	0.5110311
X3	-0.06078314	0.0232727
MSE	0.231176	
AIC	0.17511	

The mean square error of regression coefficients in the proposed model is smaller than the biased regression methods as well as the least squares regression coefficients. As supposed, the proposed model had smaller AIC value than least squares method, ridge regression and LASSO. Among these four methods, the proposed model gives the smallest AIC value.

Table 7: Comparing Least Squares, Ridge, Lasso Regression and Proposed Regression Coefficients when $n = 40$

$n = 40$								
	Least Squares		Ridge Regression		Lasso Regression		Proposed Regression	
	$\bar{\beta}_i$	S_{β_i}	$\bar{\beta}_i$	S_{β_i}	$\bar{\beta}_i$	S_{β_i}	$\bar{\beta}_i$	S_{β_i}
Intercept	0.419400	0.491641	0.378301		0.341127		0.368185	0.558804
X1	2.045913	5.500221	0.958500	2.372355	0.142900	0.242048	0.934876	0.231596
X2	1.004132	0.537591	0.995630	0.535241	0.998443	0.538235	0.955647	0.511031
X3	-0.175225	0.548361	-0.066964	0.236523	0.014500	0.024175	-0.060783	0.023272
MSE	**1.769454**		**1.04804**		**0.268153**		**0.231176**	
AIC	**0.74937**		**0.66996**		**0.18749**		**0.17511**	

CONCLUSION

It has been shown that in the presence of multicollinearity with sufficient high degrees of autocorrelation, the OLS estimates of regression coefficients can be highly inaccurate. Improving the estimation procedure is obviously necessary. Combining GLS and Ridge regression, we derived an estimator.

$$\tilde{\beta}_{GR}(k) = (X'\Omega^{-1}X + kI)^{-1}X'\Omega^{-1}Y$$

where $0 \leq k \leq 1$. $\tilde{\beta}_{GR}(k)$, though biased, is expected to perform well in the joint presence of multicollinearity and autocorrelation. However, since Ω is unknown, parameter estimates based on the biased estimator $\tilde{\beta}_{GR}(k)$ cannot be obtained in practice. Therefore, we combined Durbin's two-step method with ordinary Ridge regression to approximate those parameters. The effectiveness of our approximation can then best be examined by the Monte Carlo simulation.

The research work evealed that the proposed regression technique provides the preferred estimator in estimating all the parameters of the model based on the criteria used namely; Mean Square Error and Akaike Information Criterion (AIC) under the level of sample sizes considered. It can therefore be recommended that when the validity of other correlation assumptions cannot be authenticated, the proposed regression model cannot be used.

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