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# AN OPTIMAL DECISION MODEL FOR AMELIORATING INVENTORY ITEMS WITH STOCK DEPENDENT DEMAND

***Yusuf I. Gwanda, Falade Kazeem Iyanda and Umar Muhammad Abubakar**

Faculty of Computing and Mathematical Sciences, Kano University of Science and Technology, Wudil.-Nigeria

*Corresponding authors' email: abdusagir69@gmail.com

**ABSTRACT**

In marketplace, it is normally observed that the utility of some items increases with time. For instance, in breweries, the value of some stocked wine increases with time. In farms, the quantity or weight of fish, fast growing animals including broilers, sheep, and so on, increase with time. These phenomena are termed amelioration. In this paper therefore, we study an inventory model that determines the optimal replenishment decision for ameliorating items with stock dependent demand where the holding cost and the rate of amelioration are considered constants. The concept of differential and integral calculus was used to optimize the cost function to obtain numerical examples that illustrate the effects of parameter changes on the decision variables and to find the optimum number of replenishments. The model could be used by the organizations that deal with the relevant items.

**Keywords**: Amelioration, Deterioration, Partial backlogging, Linear demand

## INTRODUCTION

Some stored items have the property of incurring increase in quantity and/or value when kept in stock. Some domestic animals such as fishes, poultry, cattle, etc, provide demonstrative examples. It is observed in some tropical countries, that fruits and related items usually become scarce during festive and fasting periods resulting in high cost of the items. The result may not be far from the monopolistic role the merchants play in buying off most of the plantations producing the items and keeping them for months waiting for the arrival of times of festivities when the demand for the items increase exponentially. Within the period the items stay in the farm and the time they are kept in the warehouse before taken to the market, it is certain that they undergo increase in quantity or quality or both. The items that exhibit such properties are referred to as ameliorating items.

The pioneers in research on inventory used to nurse a mistaken assumption that stocked items exhibit a singular character of deterioration when in stock. They ignored or gave little attention to its ameliorative nature. It was not until in the late 1990's that Hwang [1997], for the first time studied an economic order quantity (EOQ) model and a partial selling quantity (PSQ) model in connection with ameliorating items under the assumption that the ameliorating time follows the Weibull distribution. Again, Hwang [1999, 2004] developed inventory models for both ameliorating and deteriorating items separately under the LIFO and FIFO issuing policies. Later, Moon *et al* [2005, 2006] developed an EOQ model for ameliorating/deteriorating items under inflation and time discounting. The model studied inventory models with zero-ending inventory for fixed order intervals over a finite planning horizon allowing shortages in all but in the last cycle. They also developed another model with shortages in all cycles taking into account the effects of inflation and time value of money. Later, Mondal *et al.* [2005] developed a partial selling inventory model for ameliorating items under profit maximization.

Valliathal (2016) studied the effects of inflation and time discounting on an EOQ model for time dependent deteriorating/ameliorating items with general ramp type of demand and partial backlogging rate. Valliathal (2016) studied the model under the replenishment policy, starting with shortages under two different types of backlogging rates and provided their comparative analysis.

Srivastava (2017) developed an inventory model for ameliorating/deteriorating items with trapezoidal demand and complete backlogging under inflation and time discounting. Srivastava proposed an inventory model for ameliorating/deteriorating items with inflationary condition and time discounting rate and also completely backlogged shortage.

Khatri and Gothi (2018), developed an economic production quantity inventory model for ameliorating and deteriorating items where at the inception of production activity, the constant amelioration, two parameter Weibully distributed deterioration rate and exponential demand rate was considered. At the conclusion stage of production activity, deterioration of items has been assumed to follow Pareto Type-I distribution with the same amelioration and demand rate as at the inception stage. The time dependent inventory holding cost is assumed to be a linear function of time. Shortages are acceptable and backlogged completely.

An economic order quantity model for ameliorating inventory where the lead time, the replenishment time and the demand rate are constants with no shortage of items was studied by Gwanda and Sani [2011]. The model obtained an optimum ordering quantity while keeping the relevant inventory costs minimum. Again Gwanda and Sani [2012] extended their earlier model to allow for linear trended demand.

Marchi *et al.* presented a research paper that deals with supply chain for ameliorating and deteriorating products that normally give a high sensitiveness to the surrounding environment. The model investigated the sensitivity of customers' demand to items' price and sought to coordinate the financial flow to the stock keepers and provide a set of financial schemes that optimizes accounts payable and receivable along the supply chain.

Normally, the amount maintained of most of stocked items depends on the rate at which it is consumed and the rate of its consumption is in turn observed to depend on the stocked amount. The consumption rate fluctuates with the on-hand stock level and hence large sized stores are observed to attract more customers than smaller ones. This property is also evident in most ameliorating inventories, like fishes, poultry, groceries, husbandry, and so on. In all these kinds of inventory, the demand tends to increase with theincrease in the volume of the on hand stock. It is a common knowledge that stores with larger stocks have more appeal to customers as they are most likely to provide both the quantitative and qualitative satisfaction.

Levin *et al.* [1992] observed that stores with large collection of goods are patronized more than the ones with smaller collection. These and similar observations have attracted many marketing researchers and practitioners to investigate the modeling aspects of this phenomenon. Baker and Urban [1988] have earlier focused on the analysis of inventory systems which describe the demand rate as a power function, dependent on the level of the on-hand inventory.

An inventory model for non-instantaneous deteriorating items with stock-dependent demand was developed by Wu *et al*. (2006). They considered a linearly stock dependent demand and a constant unit holding cost. In the model, shortages are allowed and the backlogging rate is variable and dependent on the waiting time for the next replenishment.

Chang *et al.* (2010) amended Wu *et al*. (2006)'s model by changing the objective to maximizing the total profit and setting a maximum inventory level in the model to reflect the facts that most retail outlets have limited shelf space. Chang *et al.* (2010) also relaxed the restriction of zero ending inventory when shortages are not desirable and provided an algorithm to find the optimal solution.

In this paper, we develop an economic quantity model for items that are simultaneously ameliorating and deteriorating with stock dependent demand and partial backlogging. The model determines the optimum cycle length so as to keep the overall costs minimum.

The proposed inventory model is developed under the following assumptions and notation:

**Assumptions**

i. The inventory system involves only one single item and one stocking point.
ii. Amelioration occurs when the items are effectively in stock.
iii. The linear stock dependent demand rate $D(t)$ at time t is assumed to be $D[(t)] = \rho + \sigma I(t)$ where $\rho$ is a positive constant, $\sigma$ is the stock dependent demand rate parameter, $0 < \sigma < 1$ and $I(t)$ is the non-negative inventory level at time t.

**Notation**

The cycle length is $T$.
The inventory carrying cost in a cycle is $C_h$
The unit cost of the item is a known constant $C$.
The replenishment cost is also a known constant $C_0$ per replenishment.
Inventory holding charge per unit i, is a known constant.
The level of inventory at any time t is $I(t)$.
The initial inventory is what enters into the inventory at $t = 0$, and it is given by $I_0$.
The amount of inventory in the interval $(0, T)$ is $I_T$.
The rate of amelioration $\alpha$ is a constant.
The ameliorated amount over the cycle $T$ when considered in terms of value (say, weight) is given by $A_T$.

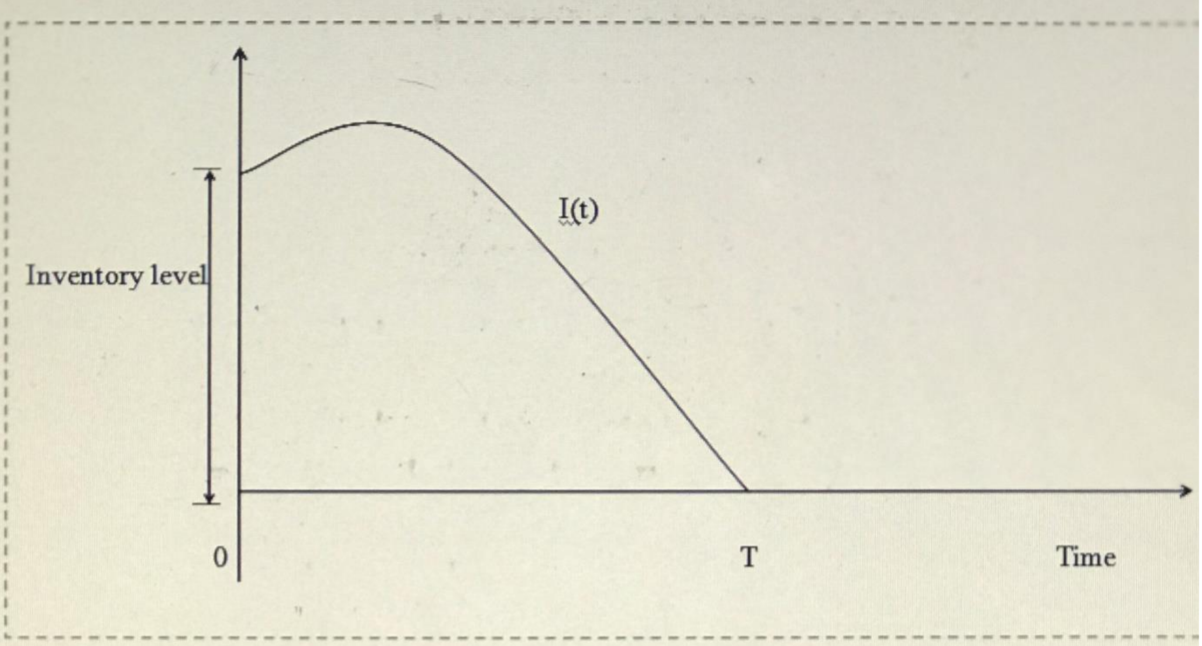


Figure 1: The graphical representation for the inventory system

## MATERIALS AND METHODS

### Model Formulation

Our objective is to determine the optimal replenishment time such that the total relevant inventory costs are kept at a minimum.

Let $I(t)$ be the on hand inventory at time $t \geq 0,$ then at time $t+\Delta t,$ the on hand inventory in the interval $(0, T_1)$ is given by:

$$I(t+\Delta t) = I(t) + \alpha . I(t)\Delta t - (\rho + \sigma I(t)).\Delta t$$

Dividing by $\Delta t$ and taking limit as $\Delta t \to 0,$ we obtain;

$$\frac{dI(t)}{dt} + (\sigma - \alpha)I(t) = -\rho, \qquad 0 \leq t \leq T \tag{1}$$

The solution of equation (1) using the integrating factor $e^{(\alpha+\sigma)t}$ is obtained as;

$$I(t) = -\frac{\rho}{\sigma-\alpha} + k_1 e^{-(\sigma-\alpha)t}, \text{ where } k_1 \text{ is a constant.} \tag{2}$$

Using $I(0) = I_0,$ we obtain the value of $k_1$ as follows:

$$k_1 = I_0 + \frac{\rho}{\sigma-\alpha}, \tag{3}$$

The value of $k_1$ is then substituted in equation (2) to get;

$$I(t) = -\frac{\rho}{\sigma-\alpha} + \left[I_0 + \frac{\rho}{\sigma-\alpha}\right]e^{-(\sigma-\alpha)t}$$

$$= -\frac{\rho}{\sigma-\alpha} + \frac{\rho}{\sigma-\alpha}e^{-(\sigma-\alpha)t} + I_0 e^{-(\sigma-\alpha)t} \tag{4}$$

Applying the boundary condition $I(T) = 0,$ we get:

$$0 = -\frac{\rho}{\sigma-\alpha} + \frac{\rho}{\sigma-\alpha}e^{-(\sigma-\alpha)T} + I_0 e^{-(\sigma-\alpha)T}$$

$$\Rightarrow I_0 = \frac{\rho}{\sigma-\alpha}(e^{(\sigma-\alpha)T} - 1) \tag{5}$$

The value of $I_0$ is now substituted into equation (4) to obtain:

$$I(t) = -\frac{\rho}{\sigma-\alpha} + \frac{\rho}{\sigma-\alpha}e^{-(\sigma-\alpha)t} + \left[\frac{\rho}{\sigma-\alpha}(e^{(\sigma-\alpha)T} - 1)\right]e^{-(\sigma-\alpha)t}$$

This gives; $$I(t) = \frac{\rho}{\sigma-\alpha}\left(e^{(\sigma-\alpha)(T-t)} - 1\right) \tag{6}$$

**Total Amount of On Hand Inventory during the Complete Cycle Time T**

This is given by:

$$I_T = \int_0^T I(t)dt$$

$$= \int_0^T \left[\frac{\rho}{\sigma-\alpha}\left(e^{(\sigma-\alpha)(T-t)} - 1\right)\right]dt$$

$$= \frac{\rho}{\sigma-\alpha}e^{(\sigma-\alpha)T_1}\int_0^T e^{-(\sigma-\alpha)t}dt - \frac{\rho}{\sigma-\alpha}\int_0^T dt$$

$$= \frac{\rho}{\sigma-\alpha} e^{(\sigma-\alpha)T}\left[-\frac{e^{-(\sigma-\alpha)T}}{\sigma-\alpha}+\frac{1}{\sigma-\alpha}\right]-\frac{\rho T}{\sigma-\alpha}$$

$$= -\frac{\rho}{(\sigma-\alpha)^2}+\frac{\rho}{(\sigma-\alpha)^2}e^{(\sigma-\alpha)T} \quad -\frac{\rho T}{\sigma-\alpha}$$

$$= \frac{\rho}{(\sigma-\alpha)^2}(e^{(\sigma-\alpha)T}-\mathrm{T}(\sigma-\alpha)-1) \tag{7}$$

**The Ameliorated Amount over the Cycle T is given by;**

$$A_T = \alpha I_T$$

$$= \frac{\alpha\rho}{(\sigma-\alpha)^2}(e^{(\sigma-\alpha)T_1}-\mathrm{T}_1(\sigma-\alpha)-1) \tag{8}$$

**The Inventory Holding Cost in a Cycle is obtained as;**

$$C_h = iCI_T$$

$$= \frac{iC\rho}{(\sigma-\alpha)^2}(e^{(\sigma-\alpha)T}-\mathrm{T}(\sigma-\alpha)-1) \tag{9}$$

**Total Variable Cost per Unit Time**

This is obtained as:

$TVC(T) = \frac{1}{T}$ {Ordering cost + Inventory holding cost per cycle + the deterioration cost per cycle – the amelioration cost per cycle }

$$\therefore TVC(T) = \frac{1}{T}\{C_0 + iCI_T - C\alpha I_T\}$$

$$= \frac{1}{T}\{C_0 + C(i-\alpha)I_T\}$$

$$= \frac{C_0}{T}+\frac{C(i-\alpha)}{T}\left[\frac{\rho}{(\sigma-\alpha)^2}(e^{(\sigma-\alpha)T}-\mathrm{T}(\sigma-\alpha)-1)\right]$$

$$= \frac{C_0}{T}+\frac{C\rho(i-\alpha)}{(\sigma-\alpha)^2 T}(e^{(\sigma-\alpha)T}-\mathrm{T}(\sigma-\alpha)-1) \tag{10}$$

$$\frac{d(TVC(T))}{dT} = \frac{d}{dt}\left(\frac{C_0}{T}\right)+\frac{d}{dt}\left(\frac{C\rho(i-\alpha)}{(\sigma-\alpha)^2 T}(e^{(\sigma-\alpha)T}-\mathrm{T}(\sigma-\alpha)-1)\right)$$

$$= -\frac{C_0}{T^2}+\frac{C\rho(i-\alpha)}{(\sigma-\alpha)^2 T^2}\left[(T(\sigma-\alpha)-1)e^{(\sigma-\alpha)T}+1\right] \tag{11}$$

For optimal cycle period $\frac{d(TVC(T))}{dT} = 0,$ that is,

$$0 = -\frac{C_0}{T^2}+\frac{C\rho(i-\alpha)}{(\sigma-\alpha)^2 T^2}\left[(T(\sigma-\alpha)-1)e^{(\sigma-\alpha)T}+1\right]$$

which simplifies to

$$0 = -(\sigma-\alpha)^2 C_0 + C\rho(i-\alpha)\left[\left(T(\sigma-\alpha)-1\right)e^{(\sigma-\alpha)T}+1\right] \qquad (12)$$

$$EOQ = I_0 = \frac{\rho}{\sigma-\alpha}\left(e^{(\sigma-\alpha)T}-1\right) \qquad (13)$$

Equation (12) can then be solved to obtain the optimum values $T^*$ of $T$ using any suitable numerical method provided that,

$$\frac{\partial^2}{\partial T^2}(TVC(T^*)) > 0, \text{That is, } \frac{2C_0}{T^3} + \frac{C\rho}{(\sigma-\alpha)^2}\left[T(\sigma-\alpha)-(\sigma-\alpha)+1\right]e^{(\sigma-\alpha)T} > 0 \qquad (14)$$

Newton-Raphson method for instance could be employed to solve the equation and obtain a solution for T. These solutions $T^*$ gives the optimal solution of (12) provided equation (14) is true.

**Numerical Examples**
Equation (12) is used to obtain the solution of the following numerical examples;

**Table 1: Input values for the five numerical examples**

| S/No. | α | ρ | C0 | C | σ | i |
|---|---|---|---|---|---|---|
| 1. | 0.45 | 2222 | 23335 | 250 | 0.34 | 0.85 |
| 2. | 0.33 | 9000 | 10098 | 300 | 0.35 | 0.23 |
| 3. | 0.10 | 2000 | 21045 | 120 | 0.13 | 0.30 |
| 4. | 0.32 | 1500 | 12230 | 300 | 0.35 | 0.42 |
| 5. | 0.22 | 6000 | 10000 | 150 | 0.40 | 0.60 |

**Table 2: Output values for the five numerical examples**

| S/No. | T* | TVC(T*) | EOQ |
|---|---|---|---|
| 1. | 195 days | 567771 Naira | 1870 units |
| 2. | 186 days | 677770 Naira | 3820 units |
| 3. | 95 days | 678905 Naira | 3344 units |
| 4. | 112 days | 567840 Naira | 4356 units |
| 5. | 88 days | 83308 Naira | 1478 units |

**Sensitivity Analysis**
Next, we carry out a sensitivity analysis to see the effect of parameter changes on the decision variables. This has been carried out by changing (that is, increasing or decreasing) the parameters by 1%, 5%, and 25% and taking one parameter at a time, keeping the remaining parameters at their original values. The results are as given in Table 3 below.

**Table 3: Sensitivity Analysis of the Fifth Example from Table 1 to See the Effects of Parameter Changes**

| Parameter | % change in parameter value | % change in the value of decision variables | | |
|---|---|---|---|---|
| | | $T^*$ | $TVC(T^*)$ | $EOQ^*$ |
| $C$ | -25 | 14 | -13 | 15 |
| | -5 | 1 | -2 | 2 |
| | -1 | -1 | -1 | 0 |
| | 1 | -1 | 1 | 0 |
| | 5 | -3 | 2 | -2 |
| | 25 | -11 | 12 | -10 |
| $C_o$ | -25 | -14 | -13 | -14 |
| | -5 | -3 | -3 | -2 |
| | -1 | -1 | 0 | 0 |
| | 1 | -1 | 0 | 0 |
| | 5 | 2 | 5 | 4 |
| | 25 | 10 | 12 | 12 |
| $\alpha$ | -25 | -8 | 7 | -6 |
| | -5 | -2 | 1 | -1 |
| | -1 | -1 | 0 | 0 |

| | | | | |
|---|---|---|---|---|
| | 1 | -1 | 0 | 0 |
| | 5 | 0 | -1 | 1 |
| | 25 | 6 | -7 | 7 |

**Table 1: Continue**

| Parameter | % change in parameter value | % change in the value of decision variables | | |
|---|---|---|---|---|
| | | $T^*$ | $TVC(T^*)$ | $EOQ^*$ |
| $\sigma$ | -50 | 0 | -1 | -1 |
| | -5 | -1 | 0 | 0 |
| | -1 | -1 | 0 | 0 |
| | 1 | -1 | 0 | 0 |
| | 5 | -1 | 0 | 0 |
| | 50 | -2 | 1 | 1 |
| $\rho$ | -25 | 13 | -13 | -13 |
| | -5 | 1 | -3 | -3 |
| | -1 | -1 | -1 | -1 |
| | 1 | -1 | 1 | 1 |
| | 5 | -3 | 2 | 4 |
| | 25 | -11 | 12 | 12 |
| $i$ | 25 | -8 | 12 | 36 |
| | -25 | 26 | -22 | 28 |
| | -5 | 3 | -4 | 5 |
| | -1 | 0 | -1 | 1 |
| | 1 | -2 | 1 | -1 |
| | 5 | -5 | 4 | -5 |

## DISCUSSION

Table 1 clearly shows that decision variables have strong effect on changes in the values of the parameters except $\sigma$ which is slightly insensitive for small changes in parameter values. We also notice the followings from the table:

i. $T^*$ increases with increase in $C_0$ and $\alpha$ but decreases with increase in $C, \rho$ and $i$ .

ii. $EOQ^*$ increases with increase in $C_0, \alpha$ and $\rho$ but decreases with increase in $C$ and $i$.

iii. $TVC(T^*)$ increases with increase in $C_0, C, \rho$ and $i$ but decreases with increase in $\alpha$.

The Table above shows that all the decision variables increase with increase in ordering cost. This is expected since if the ordering cost increases, the total cost $TVC(T^*)$ will increase and the frequency of orders will reduce so as to reduce the cost and this in effect will reduce the order quantity EOQ* and the cycle length will be shorter.

From the table also, one sees that as expected, the increase in the item's cost C, results in decrease in the EOQ and total variable cost which will invariably increase resulting in a decreased cycle period as the stockiest may not be able to stock plenty due to the prohibitive cost.

The model has provided us with interesting scenario involving its ameliorative behaviour where it conforms to the common expectation that amelioration results in accumulation of excess items and therefore longer time is required to dispose it resulting in reduced holding cost and a better *EOQ*.

The stock dependent parameters $\rho$ and $\sigma$ also conform to real life expectations where it was noticed that as both the parameters increase, the days the stock spend on the counters reduce and the *EOQ* and *TVC(T*)* increase.

## CONCLUSION

In this paper an economic order quantity model for both ameliorating items in which the demand rate is linearly dependent on inventory level has been presented. The model determines the optimal quantity to order while keeping the relevant inventory costs minimum. Numerical examples are given to illustrate the developed model and sensitivity analysis carried out on the results obtained from one of the examples in order to see the effect of parameter changes on the decision variables. The sensitivity analysis shows that all the decision variables are sensitive to changes in all the parameters.

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