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LINEAR PROGRAMMING AS DECISION MAKING TOOL FOR OPTIMAL PRODUCTION: A CASE STUDY OF YOGHURT PRODUCTION BY ATS MULTI-CONCEPT WORLDWIDE LTD IN KATSINA STATE, NIGERIA

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ABSTRACT
This research aimed at profit maximization of a production company located in Sabuwa Local government area of Katsina State, Nigeria using Linear programming model. The production data was obtained from the company after which a linear programming model was developed and solved using two different application software; The Ms Excel solver and PHP simplex. The results obtained showed a significant positive increase in profit. The analysis shows that for optimum profit based on the available raw materials used in this research, the company should produce 1,097 pieces of Y_2 yoghurt and 1,470 pieces of 50cl yoghurt per production. If the product mix is used, a profit of N 19,089.66 will be obtained by the company.

Keywords: Optimization, Operation Research, Linear Programming, Simplex Method
AMS: 80M50, 97M40, 90C05, 47N10

INTRODUCTION

The growth of manufacturing industries and the demand from consumers has made decision makers as well as researchers to develop and use strategies that will best suit the demand of the market and also produce more profit to the production companies, because profit maximization and cost minimization is the aim of every production firms. This can only be achieved by making the best use of available resources in ensuring sufficient supply to the market by the production firms. A Production firm can only survive if it is making reasonable profit from its production, therefore when a firm is able to make profit it must try and make it as large as possible, and ensure that lost and cost of production are kept at a lowest achievable percentage (Agbadudu,1999).

Profit maximization means that either a production company is producing maximum output for a given amount of input, or its uses minimum input for producing a given output. Profit maximization implies that a production firm is making an efficient allocation of its available resources in responding to competitive market condition.

Linear programming (LP) is a problem-solving approach designed to help decision makers in making good decisions proactively. The concept of linear programming was first used by G.B Dantzig in 1947 to help the Military of United State in distributing weapons and other war logistics during the Second World War.

Most researchers that undertake research on linear programming are of the view that the idea of linear programming is an indispensable tool in allocating scarce resources to provide optimal result, which is crucial to the development of production firms. The idea of linear programming was first used by the mathematician Dantzig (1947). He used linear programming to suggest the possible easiest way of distributing war logistics to the USA Armed forces at the war front. Later Dantzig advocated that the idea of linear programming can be used to solve manufacturing and industrial problems. He furthermore formulates a tool named "Simplex method" which can be used in solving linear programming problems.

This research will help the production firm to understand the optimum way of making decisions using linear programming models for optimum product-mix which will yield maximum profit to the company considering their scarce resources and production cost. This research will also highlight the relevance of linear programming models and its application in production decision making.

Omorho H.O and Ese J.I. (2018) undertake a statistical research on the impact of linear programming in production firms for some selected firms in Delta state of Nigeria. After the conclusion of their statistical analysis, they discovered that there is a strong improvement and relationship between application of linear programming model and profitability. They further posit that application of linear programming provide the best allocation of a firm limited resources which will lead to appreciable profit. They recommended that production firms should adopt this problem solving method in ensuring optimum decision making.

Igbinehi *et al.* (2015) applied linear programming in a local soap production firm that produces three different types of soap; 5g white soap, 10g red soap, 10g white soap. From their analysis they concluded that the company is using more resources on red color soap whereas they get more profit from the white soap than the red soap. They recommended that the company should produce more quantity of white soap than the red soap to obtain

profit maximization objectives.

Majeke (2013) Applied Linear programming technique and developed a Linear programming model for rural commercial farmers on the combination of crops and quantity they should be producing for optimum profit. The linear programming model he developed was solved by Excel software.

Balogun *et al.* (2012) in the research, application of linear programming was used to developed linear programming model and then solved using simplex algorithm for optimal production in the Coca-Cola Company.

Dowing (2012) suggested that the simplex method should be used for linear programming model subject to two or more constraints, graphical method for linear programming model subject to only two constraints and the Langragian method for linear programming model Subject to only one constraint.

Winston *et al.* (2012) Applied linear programming to find the best means for selling a locally made medicated soap of a soap producing company. The result of the research shows that the company should be selling one tablet of the medicated soap per pack, but the other means of selling their products i.e. 3 tablets per pack, 12 tablets per pack should be avoided

Gupta and Hira (2011) Highlighted that linear programming deal with an objective function subject to some constraint. The objective function could be profit, cost of production, production capacity, time allocation and so, on.

Adam *et al*. (2011) Advocates that linear programming model can be used to find the best feasible solution from some number of solutions by making use of step-by-step approach.

Mullan (2008) Applied linear programming and develop a linear programming model for an ice cream production company that suggest a viable cream-mix composition that will yield optimum profit to the company. The model developed during the research was solved using MS Excel Solver.

Mathews (2005) developed a linear programming model for Effective combination of nurses at Wake Forest University Baptist Medical center. At the end, the research was able to proposed optimum nursed combination from the three different type of nurses in the clinic.

Winston (2010) expressed opined that Linear programming is a mathematical modeling technique designed to make best use of firm's limited resources. Furthermore, it is a mathematical technique for optimizing a firm's limited available resources. The method of solving a linear programming problem is the simplex method. Therefore, production firms can use linear programming model in their productions so as to achieve optimum allocation of resources and to enable the firm's know the quantity and size of each product a firm should produce in order to obtain maximum profit.

Therefore, based on the ongoing studies in this area of research interest, the researchers found it worthy to apply this concept in finding the optimal production mix that effectively utilizes the available resources in Abubakar Tasi'u Sabuwa (ATS) Multi-concept Worldwide Ltd in Sabuwa, Sabuwa Local Government Area of Katsina State which are yet to adopt this model in their production decision making to enable the company maximizes its profit. With this concept, the merit of application of linear programming model to the company will be highlighted and the combination of the sizes of yoghurt produced by the company that will yield optimal profit for the company will be analyzed based on the product type available.

RESEARCH METHODOLOGY

To gather the required data for this research a formal request was made and addressed to the general manager of Abubakar Tasi'u Sabuwa (ATS) Multi-concept worldwide Ltd, requesting the breakdown of their cost analysis of yoghurt products and the ingredients involved in producing varieties of yoghurts produced by the company.

The table below is the official data received from the general manager of the company for this research.

Table 1: List of Products and the Ingredients by ATS Multi-Concept Worldwide Ltd in Sabuwa Local Government Area of Katsina State.

Ingredient/Raw Material	Products					Quantity Available
	Y_1	Y_2	Y_3	33CL	50CL	
Milk	16g	12.5g	3.5g	34.4g	52g	95955
Sugar	14g	14g	4g	26.5g	40g	76850
DVS Culture	0.004g	0.004g	0.002g	0.011g	0.016g	30.7
Potassium sorbate	0.014g	0.014g	0.008g	0.026g	0.04g	78.15
Sodium Benzoate	0.143g	0.143g	0.076g	0.265g	0.4g	792.275
Milk flavor	0.02ml	0.02ml	0.02ml	-	-	26.5
Sodium cyclamate	0	0.014g	0.069g			46.3
Nylon	2.9g	2.9g	1.8g	-	-	3182.3
Bottles	-	-	-	N15	N21	36150
Label	-	-	-	N1.5	N2.5	3675
Firewood	35kobo	35kobo	12kobo	79kobo	120kobo	224375
Labour	71kobo	71kobo	71kobo	260kobo	260kobo	705075
Diesel	100kobo	100kobo	54kobo	53kobo	53kobo	229450
Treated water	21kobo	21kobo	21kobo	21kobo	21kobo	77175
Profit	₦2.8	₦4	₦ 1	₦ 5	₦ 10	

Based on the above data in Table 1, the Linear programming Model for the yoghurt department of Abubakar Tasi'u Sabuwa (ATS) Multi concept worldwide Limited is as follows: -

Let the quantity of Y_1 yoghurt to be produced be x_1

Let the quantity of Y_2 yoghurt to be produced be x_2

Let the quantity of Y_3 yoghurt to be produced be x_3

Let the quantity of 33CL yoghurt to be produced be x_4

Let the quantity of 50CL yoghurt to be produced be x_5

Let P denote the profit to be maximized.

Then the Linear programming model for the above data is: -

Max P = $2.8x_1 + 4x_2 + x_3 + 5x_4 + 10x_5$

Subject to

$16x_1 + 12.5x_2 + 3.5x_3 + 34.4x_4 + 52x_5 \leq 95955$

$14x_1 + 14x_2 + 4x_3 + 26.5x_4 + 40x_5 \leq 76850$

$0.004x_1 + 0.004x_2 + 0.002x_3 + 0.011x_4 + 0.016x_5 \leq 30.7$

$0.014x_1 + 0.014x_2 + 0.008x_3 + 0.026x_4 + 0.04x_5 \leq 78.15$

$0.143x_1 + 0.0143x_2 + 0.076x_3 + 0.265x_4 + 0.4x_5 \leq 792.275$

$0.02x_1 + 0.02x_2 + 0.02x_3 + 0x_4 + 0x_5 \leq 26.5$

$0x_1 + 0.014x_2 + 0.069x_3 + 0x_4 + 0x_5 \leq 46.3$

$2.9x_1 + 2.9x_2 + 1.8x_3 + 0x_4 + 0x_5 \leq 3182.3$

$0x_1 + 0x_2 + 0x_3 + 15x_4 + 21x_5 \leq 36150$

$0x_1 + 0x_2 + 0x_3 + 1.5x_4 + 2.5x_5 \leq 3675$

$35x_1 + 35x_2 + 12x_3 + 79x_4 + 120x_5 \leq 224375$

$71x_1 + 71x_2 + 71x_3 + 260x_4 + 260x_5 \leq 702075$

$100x_1 + 100x_2 + 54x_3 + 53x_4 + 53x_5 \leq 229450$

$21x_1 + 21x_2 + 21x_3 + 21x_4 + 21x_5 \leq 77175$

where $x_1, x_2, x_3, x_4 and \cdot x_5 \geq 0$

The model developed based on the information provided by the company was solved using simplex method algorithm. In particular, two different application software were used to obtained the solution to the model; PHP Simplex and MS-Excel solver. PHP Simplex is an online tool to solve linear programming problems which is able to solve problems using simplex method, Two-Phase method, and Graphical method, and has no limitations on the number of decision variables nor on constraints in the problems. MS-Excel solver is inbuilt function in MS-Excel for similar purposes.

Numerical Results and Discussions

In this section, results obtained using PHP Simplex and MS-Excel Solver were presented and discussed.

Solution of the Model Using PHP Simplex

First the Number of Decision variable and the type of the constraints were inserted in the software, after which the software converted the model to standard form and generated the first simplex tableau as follows:

Table 2: First Iteration results obtained using PHP Simplex

Tableau 1			2.8	4	1	5	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Base	C_b	P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{18}	P_{19}
P_6	0	95955	16	12.5	3.5	34.4	52	1	0	0	0	0	0	0	0	0	0	0	0	0	0
P_7	0	76850	14	14	4	26.5	40	0	1	0	0	0	0	0	0	0	0	0	0	0	0
P_8	0	30.7	0.004	0.004	0.002	0.011	0.016	0	0	1	0	0	0	0	0	0	0	0	0	0	0
P_9	0	78.15	0.014	0.014	0.008	0.026	0.04	0	0	0	1	0	0	0	0	0	0	0	0	0	0
P_{10}	0	792.275	0.143	0.143	0.076	0.265	0.4	0	0	0	0	1	0	0	0	0	0	0	0	0	0
P_{11}	0	26.5	0.02	0.02	0.02	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
P_{12}	0	46.3	0	0.014	0.069	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
P_{13}	0	3182.5	2.9	2.9	1.8	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
P_{14}	0	36150	0	0	0	15	21	0	0	0	0	0	0	0	0	1	0	0	0	0	0
P_{15}	0	3675	0	0	0	1.5	2.5	0	0	0	0	0	0	0	0	0	1	0	0	0	0
P_{16}	0	224375	35	35	12	79	120	0	0	0	0	0	0	0	0	0	0	1	0	0	0
P_{17}	0	705075	71	71	71	260	260	0	0	0	0	0	0	0	0	0	0	0	1	0	0
P_{18}	0	229450	100	100	54	53	53	0	0	0	0	0	0	0	0	0	0	0	0	1	0
P_{19}	0	77175	21	21	21	21	21	0	0	0	0	0	0	0	0	0	0	0	0	0	1
Z		0	-2.8	-4	-1	-5	-10	0	0	0	0	0	0	0	0	0	0	0	0	0	0

From Table 2, the optimum solution is not reached since the last row (z- row) which contains` the objective function still has negative values. It is an indication that another iteration needs to be generated and as such the software continues with the solution of the model and generates the next tableau below.

Table 3: Second Iteration results obtained using PHP Simplex

Tableau 2			2.8	4	1	5	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Base	C_b	P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{18}	P_{19}
P_6	0	19515	16	12.5	3.5	3.2	0	1	0	0	0	0	0	0	0	0	-20.8	0	0	0	0
P_7	0	18050	14	14	4	2.5	0	0	1	0	0	0	0	0	0	0	-16	0	0	0	0
P_8	0	7.18	0.004	0.004	0.002	0.0014	0	0	0	1	0	0	0	0	0	0	-0.0064	0	0	0	0
P_9	0	19.35	0.014	0.014	0.008	0.002	0	0	0	0	1	0	0	0	0	0	-0.016	0	0	0	0
P_{10}	0	204.275	0.143	0.143	0.076	0.025	0	0	0	0	0	1	0	0	0	0	-0.16	0	0	0	0
P_{11}	0	26.5	0.02	0.02	0.02	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
P_{12}	0	46.3	0	0.014	0.069	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
P_{13}	0	3182.5	2.9	2.9	1.8	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
P_{14}	0	5280	0	0	0	2.4	0	0	0	0	0	0	0	0	0	1	-8.4	0	0	0	0
P_5	10	1470	0	0	0	0.6	1	0	0	0	0	0	0	0	0	0	0.4	0	0	0	0
P_{16}	0	47975	35	35	12	7	0	0	0	0	0	0	0	0	0	0	-48	1	0	0	0
P_{17}	0	322875	71	71	71	104	0	0	0	0	0	0	0	0	0	0	-104	0	1	0	0
P_{18}	0	151540	100	100	54	21.2	0	0	0	0	0	0	0	0	0	0	-21.2	0	0	1	0
P_{19}	0	46305	21	21	21	8.4	0	0	0	0	0	0	0	0	0	0	-8.4	0	0	0	1
Z		14700	-2.8	-4	-1	1	0	0	0	0	0	0	0	0	0	0	4	0	0	0	0

Also, Table 3 shows similar pattern as Table 2 which is an indication that optimum solution has not been reached and therefore, further iteration is required. Hence the software generates the next simplex tableau below:

Table 4: Third Iteration results obtained using PHP Simplex

Tableau 3			2.8	4	1	5	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Base	C_b	P_0	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_{15}	P_{16}	P_{17}	P_{18}	P_{19}
P_6	0	5797.32759	3.5	0	-4.258620	3.2	0	1	0	0	0	0	0	0	-4.310	0	-20.8	0	0	0	0
P_7	0	2686.20690	0	0	-4.689655	2.5	0	0	1	0	0	0	0	0	-4.828	0	-16	0	0	0	0
P_8	0	2.79034483	0	0	-0.000482	0.0014	0	0	0	1	0	0	0	0	-0.0014	0	-0.01	0	0	0	0
P_9	0	3.98620690	0	0	-0.000690	0.002	0	0	0	0	1	0	0	0	-0.00483	0	-0.016	0	0	0	0
P_{10}	0	47.3448279	0	0	-0.012759	0.025	0	0	0	0	0	1	0	0	-0.04931	0	-0.16	0	0	0	0
P_{11}	0	4.55172414	0	0	0.007586	0	0	0	0	0	0	0	1	0	-0.00690	0	0	0	0	0	0
P_{12}	0	30.936207	-0.014	0	0.060310	0	0	0	0	0	0	0	0	1	-0.00483	0	0	0	0	0	0
P_2	4	1097.4138	1	1	0.6206897	0	0	0	0	0	0	0	0	0	0.3448	0	0	0	0	0	0
P_{14}	0	5280	0	0	0	2.4	0	0	0	0	0	0	0	0	0	1	-8.4	0	0	0	0
P_5	10	1470	0	0	0	0.6	1	0	0	0	0	0	0	0	0	0	0.4	0	0	0	0
P_{16}	0	9565.517	0	0	-9.724138	7	0	0	0	0	0	0	0	0	-12.07	0	-48	1	0	0	0
P_{17}	0	244958.62	0	0	26.931034	104	0	0	0	0	0	0	0	0	-24.48	0	-104	0	1	0	0
P_{18}	0	41798.62	0	0	-8.068966	21.2	0	0	0	0	0	0	0	0	-34.48	0	-21	0	0	1	0
P_{19}	0	23259.21	0	0	7.965517	8.4	0	0	0	0	0	0	0	0	-7.241	0	-8.4	0	0	0	1
Z		19089.66	1.2	0	1.482759	1	0	0	0	0	0	0	0	0	1.3793	0	4	0	0	0	0

Finally, Table 4 shows no negative values in the last row which is the objective function row and this is an indication that the optimum result for the given model is reached, i.e the maximum profit is obtained and no further iteration is required because even if done the same result will be generated. This show that optimum solution for the given model is obtained after four iterations.

Observed that from the first column of tableau, p_1 represents x_1, p_2 represents x_2, p_3 represents x_3, p_4 represents x_4, and p_5 represents x_5. Hence for optimal profit the company needs to produce the following quantity of yoghurts.

x_1=Y_1=0
x_2=Y_2=1097
x_3=Y_3=0
x_4=33cl=0
x_5=50cl=1470

Furthermore, observed from the first column only p_2 and p_5 are visible showing that p_1 , p_3, and p_4 are all zero quantities and corresponding quantities of p_2 and p_5 as well as the profit (Z) are given in the third column as indicated above. And if the production mix is use, a maximum profit of ₦ 19,089.6551 will be obtained by the company.

Solution of the Model Using Ms-Excel Solver

The second software used to find the solution of the model is Ms-Excel solver, below is the built in report generated by the software after the data of the model was inputted into the excel spreadsheet and relevant formulas were stored.

Table 5: Optimal solution obtained using MS-Excel Solver

Microsoft Excel 12.0 Answer Report
Worksheet: [Book1JABIR.xlsx]Sheet1
Report Created: 31/Aug/19 3:01:47 PM

Target Cell (Max)

Cell	Name	Original Value	Final Value
I5	value OBJECTIVE VALUE	22.8	19089.3793

Adjustable Cells

Cell	Name	Original Value	Final Value
D5	value x1	1	0
E5	value x2	1	1097.34483
F5	value x3	1	0
G5	value x4	1	0
H5	value x5	1	1470

From Table 5, after inputting the corresponding values from the model into the Excel worksheet the above values were obtained where the Profit (objective function value) of the mix production is ₦ 19089.3793 which requires that $x_1 = x_3 = x_4$ = zero quantities with $x_2 = 1097$ and $x_5 = 1470$ quantities. And this is in agreement with the solution obtained from the earlier method used.

CONCLUSION

The production data was obtained from the company after which a linear programming model was developed and solved using two different application software; The Ms-Excel solver and PHP simplex. The results obtained showed a significant positive increase in profit. The analysis shows that for optimum profit based on the available raw materials used in this research, the company should produce 1,097 pieces of Y_2 yoghurt and 1470 pieces of 50cl yoghurt per production. If the product mix is used, a profit of ₦ 19,089.66 will be obtained by the company.

Recommendations to the Company

Based on the findings of this research, the following recommendations are made: that the management of Abubakar Tasi'u Sabuwa (ATS) worldwide Ltd should use the result of this research for profit maximization in future productions, Abubakar Tasi'u Sabuwa Multi concept worldwide Ltd should adopt the concept of linear programming in all their production departments for production decision making, to enable them achieve maximum profit. Because, the linear programming model yields a better solution, higher profit and accelerate the achievement of the organization goals more than the mere assumptions and Manufacturing industries should employ trained and competent operation research personnel, organize workshops to create awareness of the concept of Linear programming and adopt the concept of Linear programming in their production decision making.

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