APPLICATION OF MULTI-ITEMS EPQ MODELS IN COURSE MATERIALS MANAGEMENT: TOWARDS BUILDING A DECISION SUPPORT SYSTEM FOR THE NATIONAL OPEN UNIVERSITY OF NIGERIA

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ABSTRACT

The classic multi-item Economic Production Quantity (EPQ) model was applied in this research to solve the Course Materials production problem of National Open University of Nigeria (NOUN). The EPQ model is one of the essential components of the proposed Decision Support System (DSS) for production and inventory management of course materials in NOUN. Python programming was used to develop the DSS to give a flexible interface that is easy to use by all users. The EPQ module of the DSS generated essential outputs which answers basic questions such as when to produce, how much of each item to produce per production run, the cost of producing each item and the interval between the productions run. Testing of the EPQ module was carried out using test data. When implemented, the proposed solution will enhance decision making in the area of Course Material production at NOUN printing press and reduce the difficulties encountered by staff when using manual processes to resolve complex models on regular basis.

Keywords: Course Materials Production, Multi-Item Economic Production Models, EPQ, Multi-item Production Cycle, Python Programming

INTRODUCTION

Using human intuitive judgment and manual computations to manage complex Production processes, is undoubtedly daunting with too many associated challenges. More so, these manual processes most times produce unreliable outputs which may lead to wastages, extra costs and other disadvantages compared with automated systems. Production management problems exist in many organizations where inventory items are needed for day to day running of the organizations. Such problems are further compounded when multi-item inventories are required at varying quantities. The basic problems are usually to determine: how much quantity of each item to produce, the number of production batches required each year, the optimum interval between productions and the inventory cost for a production run.

A study of the system of Course Materials production planning and inventory management at the National Open University of Nigeria shows that a lot of problem-solving and decision-making processes are manually carried out. This has effect on the overall productivity and efficiency of the system as evident in the non-availability of some Course Materials at the Study Centres when demanded by students. The Management of NOUN are faced with the challenge of managing over One Thousand Eight Hundred (1800) different Course Materials (CM), which they are obligated to print and distribute to students every semester. This challenge is further compounded by the growing number of students and the new courses introduced by NOUN on regular basis. There is need therefore, to develop and implement a comprehensive Decision Support System for problem solving across the entire Course Materials production and inventory management processes in NOUN. Adoga et al., (2022) discussed a forecast model to determine the quantities of each Course Material needed to meet students' demand within a forecast year and applied inventory models to manage orders and other inventory control activities. This research presents EPQ models that will help Decision Makers in making the right decisions whenever there is need to produce the Course Materials at the University's Printing Press or any other production environment in general. The EPQ model forms part of the model base of a proposed computer based DSS that will aid in reducing the deficiencies of human intuitive judgment, reduce stress resulting from regular manual computations of complex models by staff and finally aid in quality decision making. A prototype of the EPQ module of the DSS is also discussed in this paper.

Economic Production Quantity Models

Moon et al., (2002) defined Economic Lot Scheduling Problem (ELSP) as the problem of finding the production sequence, production times and idle times of several products in a single machine on a repetitive basis to ensure demands are made without stock-outs or backorders and average inventory holding and setup costs are minimized. Talapatra et al., (2016) presented an Economic Lot Scheduling for time varying demand with Stock out which provided a production schedule of a set of items in a single machine to minimize the long run average holding and set up cost. The work was based on assumptions of time varying demand and production rates with stock out allowed. Khouja & Mehrez (1994) extended the economic production lot (EPL) size model to cases where the production rate is a decision variable. For both weak and strong relationships between the production rate and process quality, results indicated that the optimal production rate may be considerably different from the production rate that minimizes unit production cost. Zipkin (1991) considered ELSP in which items may be produced several times in different amounts during a cycle. The paper used the method of parametric quadratic program combined with other Economic Order Quantity computations to obtain the optimal lot sizes and cycle length. Haessler & Hogue (1976) developed a procedure for automatically generating feasible solutions for the single-machine multi-product lot scheduling problem. The model checks for solution feasibility and adjusts the fundamental cycle time and production frequencies in a systematic manner until a feasible solution is obtained. Bukhari (2011) focused on a continuous time model of a production maintenance system in which a firm sells some of the produced single product and stock the remaining. Results showed that the actual production rate exponentially tend to

the goal value while the inventory level, deterioration rate, and desired production rate oscillate about their goal values. Cunha et al., (2017) considered the multi-item ELSP with remanufacturing and "uncapacitated" production. Feasible solutions and bounds were obtained by solving independent single item problems separately.

Hwang (2010) integrated production and transportation operations into the ELSP with the objective of achieving a cost effective production and shipment schedule. Mokhlesiana et al. (2010) presented an ELSP model with the consideration of the time value of money. Results showed that the total cost of the proposed model is reduced, significantly when the model is applied. Chang et al., (2013) also presented a model that adds the raw materials stock-holding cost to the EPQ model and takes the time value of money into consideration. Carvalho & Nascimento (2016) focused on the lot sizing of companies with multiple plants where each plant has finite planning horizon. The paper introduced a novel Lagrangian heuristics for solving benchmark instances. Tang et al., (2020) considered a single item dynamic ELSP with production or outsourcing decisions. Results indicated that total cost can be reduced through backlogging or outsourcing. Krishnamoorthi & Panayappan (2012) proposed an EPQ model to assist manufacturers and retailers in determining the optimal quantity, cycle time and inventory total cost. The result of the model was thereafter coded using Microsoft Visual Basic 6.0. Ben-Dayal & Hariga (2000) modeled the effects of imperfect production processes with assumption that the production facility starts in the in-control state where items of high or perfect quality are produced. A case where the facility may deteriorate with time resulting to nonconforming production was also considered. Bai et al., (2010) presented an ELSP with perishable inventory and economies of scale cost functions. The model used shift technique to obtained feasible solution and takes the optimal solution of the sub-problem as an approximate solution of the problem.

Cheng (1991) proposed an equation to model the relationship between unit production cost and process capability and quality assurance expenses for the EPQ problem. The optimal solution was derived using differential calculus, which yields a simple closed form expression for the optimal value of both production quantity and expected fraction acceptable. Gutierrez et al., (2021) proposed an efficient algorithm for the ELSP with storage capacity. The model uses a geometric technique to speed up the algorithm for a class of sub-problems generated by dynamic programming, which can be solved in linearithmic time. Su & Lin (2013) introduced a model that adds the stock-holding cost of raw materials to the EPQ model. The approach established two more useful models with the optimal cycle lengths from each of the models computed.

Decision Support System

Marakas (2003) defined decision support system as a system under the control of one or more decision makers, that assists in the activity of decision making, by providing an organized set of tools intended to impose structure on portions of the decision-making situation and to improve the ultimate effectiveness of the decision outcome. Researchers have identified four core components required in building DSS namely the: User Interface, Database, Models and Analytical tools, and lastly, Architecture and Network (Sprague & Carlson, 1982 cited in Power, 2015). Hasan et al., (2016) and Marek & Roger (2010) in their separate works discussed three essential components of DSS namely; Data Management Subsystem, Model Base Management Subsystem, and Dialog Generation and Management Subsystem also known as user interface subsystem. Marakas (2003) discussed two additional components of a DSS; Knowledge engine and the Users.

Parekh et al., (2008) developed an Integrated Inventory Ranking System for Inventory Management. The resultant system is useful in solving the problem of subcontracting vs. manufacturing locally. Arica et al., (2014) proposed a framework that can be utilized to design or enhance decision support systems for scheduling activities in complex and uncertain manufacturing environments. Giachetti (1997) described a prototype material and manufacturing process selection system that integrates a formal multi-attribute decision model with a relational database. The decision model enables the representation of the designer's preferences over the decision factors. Cecilia et al., (2011) presented a framework of DSS for inventory management which allows companies to specify all the model requirements for inventory control, to enable them achieve better results. Gustriansyah et al., (2015) developed a decision support system for inventory management in pharmacies, to predict inventory using Fuzzy Analytic Hierarchy Process and Sequential Pattern Analysis. Jiang (2020) developed an internet of Things based decision-making system to provide solution for intelligent and collaborative production and distribution to meet individual needs. Shirazi et al. (2010) presented a simulation based intelligent decision support system for real time control of a flexible manufacturing system with machine and tool flexibility.

MATERIALS AND METHODS

Here we present the steps followed in applying the classic multi-items economic production quantity models developed by Sharma (2005) to the Course Material inventory data of the National Open University of Nigeria. The steps followed in developing the EPQ module of the DSS is also presented in this section.

The EPQ Model

In the Production and Consumption EPQ Model, the production lots are assumed to arrive continuously at a production rate K, unlike the classic Economic Order Quantity (EOQ) model where lots arrive instantaneously. This happens when a production process at rate K feeds the inventory at a lesser demand rate R. Since the materials produced are used to service demand, the maximum inventory level Q is never attained. When a production run ends, the produced inventory items are issued at a demand rate R, until it reaches 0 at the end of the cycle as shown in figure 1 Sharma (2005). The figure shows the cycle of a single inventory item, but there are many cases where multiple items are required to run an organization like the case of NOUN Course Materials. These multiple items compete for time allocation and other resources when produced in a single machine with limited production capacity. The multi-items EPQ model is therefore useful in finding the optimum production schedules of several products produced in a single machine so as to meet the demands without stock-outs.
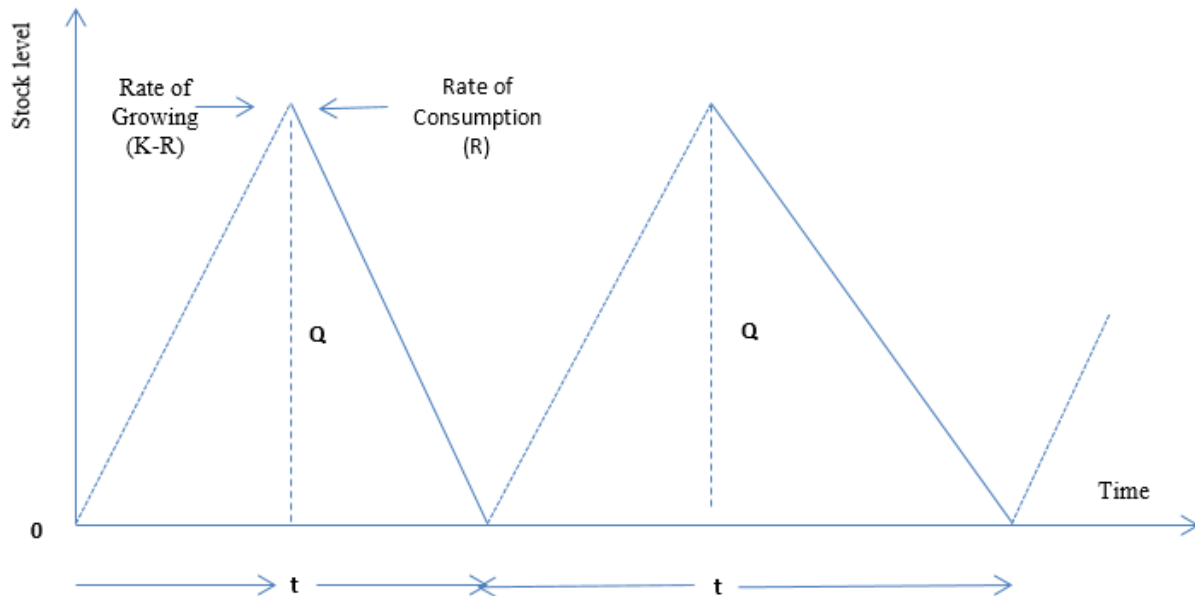
Figure 1: The Production and Consumption EPQ Model with finite replenishments (Source: Sharma, 2005).

The classic EPQ model is given by:

$$Q = \left(\frac{q}{K}\right)(K - R) \tag{1}$$

$$C(q) = \frac{1}{2}q\left(\frac{K-R}{K}\right)C_1 + \left(\frac{R}{q}\right)C_3 = \text{cost equation} \tag{2}$$

$$q^* = \sqrt{\frac{2RC_3}{C_1\left(1-\frac{R}{K}\right)}} = \text{Optimal lot size formula} \tag{3}$$

$$t^* = \frac{q^*}{R} = \sqrt{\frac{2C_3}{C_1 R\left(1-\frac{R}{K}\right)}} = \text{Optimal time interval} \tag{4}$$

$$N = \frac{R}{q^*} \quad = \text{Number of cycles per year} \tag{5}$$

For a single product produced on the machine, N is expressed in the form:

$$N = \sqrt{\frac{RC_1\left(1-\frac{R}{K}\right)}{2C_3}} \tag{6}$$

For several products manufactured in a single machine, N is given by:

$$N = \sqrt{\frac{\sum_{i=1}^{n}\left[R_i C_1^{(i)}\left(1-\frac{R_i}{K_i}\right)\right]}{2\sum_{i=1}^{n} C_3^{(i)}}} \quad , i = 1,2\ldots, n. \tag{7}$$

Notations
n = Number of items.
R_i = Demand rate for the ith item (i= 1,2..,n).
K_i = Production rate of the ith item
$C_1^{(i)}$ = Holding (or carrying) cost per unit of the quantity of ith item.
$C_3^{(i)}$ = Setup cost per production run.
C(q) = Inventory Cost at EPQ (cost of an item per production run).
TC(q) = Cost of all inventory items at EPQ.
Q = Maximum stock level

q_i^* = Optimal lot size for i^{th} item (The number of ith item produced per production run).
t^* = Optimal time interval between production cycles.
N = Number of cycles per year (i.e total number of production runs in a year).

Assumptions of the Models
  i. Production is instantaneous with no lead time.
  ii. Shortages are not permitted
  iii. K is finite, uniform and greater than R.

Applying the EPQ Model to the multi-item Course Material inventories of NOUN
The core objectives of the EPQ model are to help determine the:
  i. Economic lot size (q*) for each item.
  ii. Number of production batches per year (N).
  iii. Optimum time interval between productions (t*).
  iv. Total cost of inventories per unit time of production.

The following steps were followed in applying the EPQ model to the data of NOUN.
i. Import or enter the model inputs for each item (R, C_1, C_3 and K)
ii. Determine the optimum production batches (N), by applying equation 7. i.e

$$N = \sqrt{\frac{\sum_{i=1}^{n}\left[R_i C_1^{(i)}\left(1-\frac{R_i}{K_i}\right)\right]}{2\sum_{i=1}^{n} C_3^{(i)}}} \quad , i = 1,2\ldots, n.$$

iii. Determine optimal lot size q_i^* for each item. Having determined N, we rewrite equation 5 as:

$$q_i^* = \frac{R_i}{N} \quad , \text{for } i=1,2,\ldots,n$$

iv. Determine the Optimal time interval between production cycles (t*). This can be computed from

equation 4 or $t^* = \sqrt{\dfrac{2\sum_{i=1}^{n} C_3^{(i)}}{\sum_{i=1}^{n}\left[R_i C_1^{(i)}\left(1-\frac{R_i}{K_i}\right)\right]}}$, i = 1,2…, n. (8)

v. Determine the total inventory cost C(q) per production run using Equation 2 or

$$C(q) = 2 * \left(\frac{R_i}{q_i^*} * C_3^{(i)}\right) \text{ for each item.}$$

vi.   TC(q)= $\sum_{i=1}^{n}$ C(q) can be easily computed if required.

Overview of the Proposed Decision Support System
Table 1 shows components of the proposed DSS. The design consists of Data Management Subsystem, Model Base Management Subsystem, Dialog Generation and Management Subsystem and the Users of the system. Details of the design and implementation of all the components of the DSS are presented in Adoga et al., (2022). A module is developed for each model in the model base of the DSS. The EPQ module was designed to execute the EPQ model discussed in this paper while the forecast and inventory modules were designed to predict students' enrolment and manage inventory respectively.

Implementation
Jupyter notebook provided the flexible IDE used in the development of the DSS using python programming language. The Python pandas library was used to handle the large data used in the design. The maths and sklearn.metrics libraries provided tools for coding of the EPQ and other mathematical and statistical models while the Tkinter library was used to provide a flexible Graphical User Interface (GUI). A section of the code is shown in Appendix 1.

Testing and validation of the developed EPQ module was carried out using the input shown in table 2. The EPQ module receives its list of inventory items and their respective annual forecast demands from the forecast module of the DSS. The setup costs, holding costs and annual production rates are user defined variables supplied by experts and production managers in a Microsoft Excel sheet and imported into the python designed GUI for output processing.

Table 1: Components of the DSS

S/N	Component	Sub-components	Definition and function
1	Data Management Subsystem	Tables and Queries	The tables collects and stores data while the queries are used for manipulating entries in the database
2	Model Base Management Subsystem	• The Holt Winters Additive forecast model • The Study pack algorithm • The Inventory Management models • The EPQ model	Provides all the necessary support for the execution of the model libraries
3	Dialog Generation and Management Subsystem	• The Home Screen • The Forecast Model Menu • The Inventory Model Menu • The Production Model Menu • The file Menu • The Study Pack menu • The Tools menu	The user interfaces of the DSS used for execution of the models and other functions of the DSS
4	Users	• Staff of the Management Information System unit • Departmental Registration Officers • Staff of the Procurement unit • Staff of the Store Unit • Staff of Technical Unit	The human aspect which drives other aspect of the system through application of knowledge and performance of assigned roles.

Table 2: Inputs of the multi-item EPQ model

Course code (Inventory item)	Annual Forecast Demand (R_i)	Setup Cost in Naira per Unit item (C_3)	Holding Cost in Naira per unit item (C_1)	Annual production rate of ith item (K_i)
CIT101	1506	400	200	2259
CIT102	1297	400	200	1945.5
CSS111	1342	350	200	2013
CSS112	1297	350	200	1945.5
CSS121	1342	350	200	2013
CSS132	1297	350	200	1945.5
CSS133	1342	350	200	2013
CSS134	1297	350	200	1945.5
CSS136	1297	350	200	1945.5
CSS152	1297	350	200	1945.5
ECE110	273	300	200	409.5
ECE112	273	300	200	409.5
ECE113	271	300	200	406.5
ECE120	273	300	200	409.5

ECE121	271	300	200	406.5
ECE123	271	300	200	406.5
ECO121	1342	400	200	2013
EDU111	435	300	200	652.5
EDU112	441	300	200	661.5
EDU114	441	300	200	661.5
ENG111	164	400	200	246
ENG113	164	400	200	246
ENG114	168	400	200	252
ENG121	164	400	200	246
ENG122	168	400	200	252
ENG141	164	400	200	246
ENG162	168	400	200	252
ENG172	168	400	200	252
GST101	1777	350	200	2665.5
GST102	1738	350	200	2607
GST105	1777	350	200	2665.5
GST107	1777	350	200	2665.5
PCR111	1342	400	200	2013
PCR114	1297	400	200	1945.5
PED112	273	300	200	409.5
PED122	271	300	200	406.5
PED130	273	300	200	409.5
PED144	271	300	200	406.5
POL111	1342	400	200	2013
POL126	1297	400	200	1945.5

RESULTS AND DISCUSSIONS

Table 3 shows the output of the EPQ model. The values of EPQ (q_i*) of each item shown in table 3 represents the optimum number of items required per production run, so as to meet the projected yearly demand without stock out . C(q) represents the cost of producing the optimum production quantities of each item per production run. The summary portion at the bottom of table 3 also shows the values of TC(q) = 246681.7, N= 8.747576 and t* = 0.114317 years (41.73 days) obtained for the given illustration.

These results provide answers to basic question such as when to produce, how much to produce and the cost of each production. For every production run, the EPQ (q_i*) of each item is produced. When the EPQ (q_i*) is produced, it will service demand for 41.73 days before another production is expected to take place. A total of 8.75 productions are required per year with a total cost of ₦246681.7 per production run. The results are in line with the production and consumption cycle shown in figure 1 and the basic objectives of the EPQ model outlined above. Production stops when the EPQ of each item is produced and the available stocks are consumed at a constant demand rate until the stock gets to the minimum level when the next production begins.

Figure 2 shows the interface of the EPQ module of the developed DSS prototype. The interface consists of an input window which displays the model inputs imported by the user, a tool for executing the EPQ model and an output window which displays the outputs of the system. In the design, a user selects the production model menu to begin execution of the EPQ model. The interface then provides a filedialog to enable the user locate and import an input file which must have been prepared and saved in the client computer before the execution time. The chosen input is then displayed in the input window at the top of the GUL as shown in figure 2. The user is allowed to preview the displayed inputs before executing the EPQ model by clicking on the execution tool at the top left hand corner. The outputs generated are then displayed in the output window at the bottom as shown in figure 2. The outputs can thereafter be exported to a csv or Microsoft Excel file for further processing as shown in Table 3.

Table 3: Outputs of the multi-item EPQ model

Course code (Inventory item)	EPQ (q_i*)	C(q) (in Naira)
CIT101	172.162	6998.061
CIT102	148.2696	6998.061
CSS111	153.4139	6123.304
CSS112	148.2696	6123.304
CSS121	153.4139	6123.304
CSS132	148.2696	6123.304
CSS133	153.4139	6123.304
CSS134	148.2696	6123.304
CSS136	148.2696	6123.304
CSS152	148.2696	6123.304
ECE110	31.20864	5248.546
ECE112	31.20864	5248.546
ECE113	30.98001	5248.546
ECE120	31.20864	5248.546
ECE121	30.98001	5248.546
ECE123	30.98001	5248.546

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ECO121	153.4139	6998.061
EDU111	49.72806	5248.546
EDU112	50.41396	5248.546
EDU114	50.41396	5248.546
ENG111	18.74805	6998.061
ENG113	18.74805	6998.061
ENG114	19.20532	6998.061
ENG121	18.74805	6998.061
ENG122	19.20532	6998.061
ENG141	18.74805	6998.061
ENG162	19.20532	6998.061
ENG172	19.20532	6998.061
GST101	203.142	6123.304
GST102	198.6836	6123.304
GST105	203.142	6123.304
GST107	203.142	6123.304
PCR111	153.4139	6998.061
PCR114	148.2696	6998.061
PED112	31.20864	5248.546
PED122	30.98001	5248.546
PED130	31.20864	5248.546
PED144	30.98001	5248.546
POL111	153.4139	6998.061
POL126	148.2696	6998.061
Cost of all inventory items at EPQ	=	246681.7
The total number of production runs in a year (N)	=	8.747576
Optimal time interval between production cycles (t*)	=	0.114317
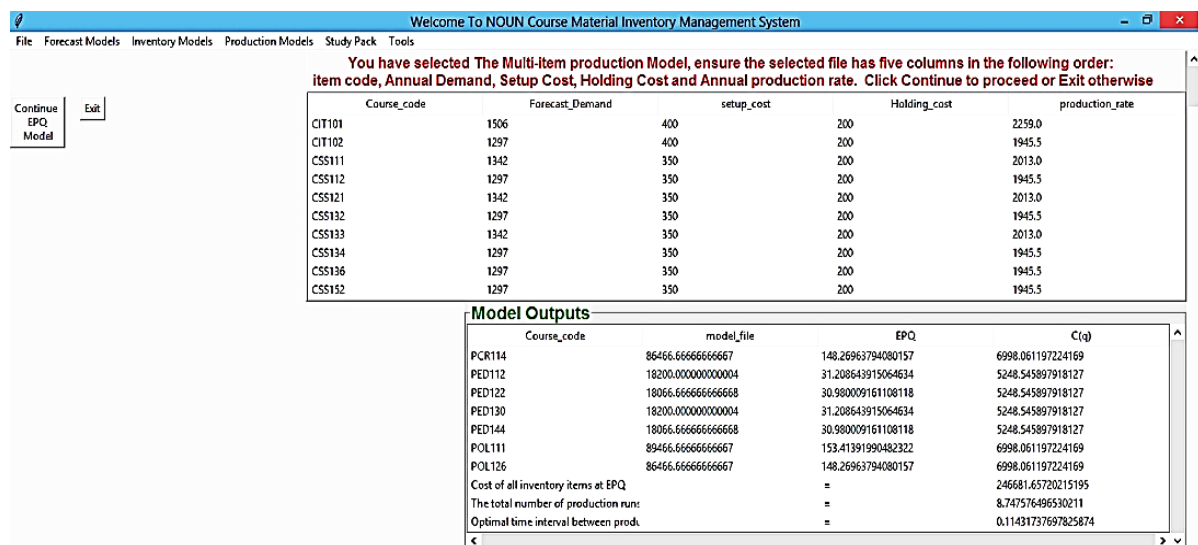
Figure 2: Input and output of the EPQ module.

CONCLUSION

The classic multi-item Economic Production Quantity model was applied in this research to solve the Course Materials production problem of NOUN. The model was built into a Decision Support System designed for inventory and production management of course materials in NOUN. Jupyter notebook provided the flexible Integrated Development Environment (IDE) used in the development of the DSS using python programming language. Testing and validation of the EPQ module was carried out and result showed high accuracy and speed of the developed system as compared with manual computations.

Effective modeling of the production management system of Course Materials in the National Open University of Nigeria will solve the problem of non-availability of these important study materials when demand is made for them by students. Also automating the processes involved in the production of these materials will enable staff and Managers to carry out their duties with minimal stress and avoid relying heavily on human intuitive judgment in decision making. Application of the model discussed in this work will significantly increase productivity, efficiency and give management enough time to budget and plan ahead towards meeting the demands for course materials and other services in a forecast year. Finally, the results of this research will breach significantly the dearth of knowledge in the area of application of model-driven Decision Support Systems in problem solving in Nigerian institutions.

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Appendix 1: Python codes

```python
from tkinter import ttk
import pandas as pd
import math
def cont_epq():
    global N
    #global sa2
    global sum_qi
    data=[]
    global dfspace
    global dfspace1
    dfspace = pd.DataFrame()
    for i in range(len(df.index)):
        #N=math.sqrt(df.Forecast_Demand.iloc[i]*df.Holding_cost.iloc[i]*(1-
(df.Forecast_Demand.iloc[i]/df.Annual_production_rate.iloc[i])/df.Holding_cost.iloc[i])
        n_num = df.Forecast_Demand.iloc[i]*df.Holding_cost.iloc[i]*(1-
(df.Forecast_Demand.iloc[i]/df.production_rate.iloc[i]))
        data.append(n_num)
    #dfspace = pd.DataFrame(data,data1)
    n_dem = 2*df['setup_cost'].sum()
dfspace['Course_code'] = df.Course_code.iloc[0:]
dfspace['model_file'] = data
    sum_n_num=dfspace['model_file'].sum()
    N = math.sqrt (sum_n_num/n_dem)
    t = math.sqrt (n_dem/sum_n_num)
    print("The total number of production runs in a year (N) = ", N)
    print("Optimal time interval between production cycles (t in years) = ", t)
    for i in range(len(df.index)):
        qi = df.Forecast_Demand.iloc[i]/N
        cq = 2*(df.Forecast_Demand.iloc[i]/qi)*df.setup_cost.iloc[i]
        data1.append(qi)
        data2.append(cq)
    dfspace['EPQ'] = data1
    dfspace['C(q)'] = data2
tc=dfspace['C(q)'].sum()
print("Cost of all inventory items at EPQ = ", tc)
    summary_row = pd.DataFrame({'Course_code':['Cost of all inventory items at EPQ ','The total number of production
runs in a year (N)', 'Optimal time interval between production cycles (t in years)'], 'model_file':["", "", ""],'EPQ':[' = ', ' = ',
' = '],'C(q)':[tc,N,t]})
    dfspace1 = dfspace.append(summary_row, ignore_index = True)
    y=displayoutput()
win.mainloop()
```