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A MATHEMATICAL MODEL FOR DIABETES MANAGEMENT

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ABSTRACT

Treating and controlling of diabetes is called diabetes management. The management of diabetes can be done through improvement of the patient dietary knowledge, attitudes, and practices. This research work is aimed at introducing dietary (diet restriction) to an earlier model for the detection and control of diabetes: to study the effect of physical exercise and dietary on excess glucose, insulin and epinephrine concentrations in the blood in managing diabetes. The resulting model is found to be uniformly and asymptotically stable. The result shows that when physical exercise is combined with recommended dietary in the management of diabetes, the excess glucose, insulin and epinephrine concentrations returns to their normal level faster with time compared to when only physical exercise is used as a control measure. Hence, it is recommended that in the management of diabetes, both physical exercise and dietary should be used as control measures.

Keywords: Diabetes, Dietary, Epinephrine, Glucose, Insulin, Physical Exercise

INTRODUCTION

There is no cure for diabetes, but it can be treated and controlled which is called diabetes management. The goals of managing diabetes are to: keep blood glucose levels as near to normal as possible by balancing food intake with medication and activity, maintain blood cholesterol and triglyceride (lipid) levels as near the normal ranges as possible, control blood pressure, decrease or possibly prevent the development of diabetes-related health problems (American Diabetes Association, 2018; 2020). Diabetes can be controlled through improvement in patient's dietary knowledge, attitudes, and practices (Sami *et al*., 2017).Unhealthy eating habits and physical inactivity are the leading causes of diabetes (Berckman, 2006). The management of diabetes can be done by;planning what you eat and following a balanced meal plan, exercising regularly, taking medication, if prescribed, and closely following the guidelines on how and when to take it, monitoring your blood glucose and blood pressure levels at home, keeping your appointments with your healthcare providers and having laboratory tests completed as ordered by your doctor (American Diabetes Association, 2020).

Several studies have been done on the dynamic and management of diabetes. Bolie (1961), Ackerman and McGucking (1964) and Bergman *et al*. (1979) worked on models that gives better understanding of the glucose – insulin regulatory. An improvement of the earlier model was done by Kwach *et al*. (2011) by introducing a third variable called the Epinephrine. This model gives a better understanding of the glucose – insulin – epinephrine regulatory system based on Glucose Tolerance Test (GTT) and can be used for detection of diabetes using the sample collected from GTT. Kwaghkor and Luga (2016) extended the Kwach *et al*. (2011) model by in-cooperating physical exercise as a diabetes control measure. The results reviewed that physical exercise can control diabetes by bringing the excess glucose, insulin and epinephrine concentrations in the blood to normal level with time.This study is aimed at introducing dietary (diet restriction) to the model by Kwaghkor and Luga (2016) to study the effect of physical exercise and dietary on excess glucose, insulin and epinephrine concentrations in the blood assuming that, the normal daily calorie intake for a diabetic per day isaround1500 to 1800 calorie (MayoClinic, 2019).

MATERIALS AND METHODS

The model by Kwaghkor and Luga (2016) was based the following parameters, variables and assumptions.

Parameters of the Existing Model

The parameters of the existing presented as follows;

$\boldsymbol{a}$: The rate of glucose consumption in tissues,

$\boldsymbol{b}$: The rate at which insulin facilitates the uptake of glucose by the body tissues,

$\boldsymbol{c}$: The rate at which insulin is secreted in the body by the pancreas,

$\boldsymbol{d}$: The rate at which insulin is used up in the body tissues,

$\boldsymbol{f}$: The rate at which epinephrine inhibits glucose uptake by muscle tissues,

$\boldsymbol{k}$: The rate at which epinephrine acts on the pancreas to inhibit insulin secretion,

$\boldsymbol{l}$: The rate of change in plasma epinephrine concentration as a result of low glucose concentration,

$\boldsymbol{m}$: The change in plasma epinephrine concentration as a result of low insulin concentration,

$\boldsymbol{n}$: The rate at which epinephrine is secreted in the blood.

$\boldsymbol{p}$: The rate at which glucose concentration is used up due to physical exercise,

$\boldsymbol{q}$: The rate at which insulin concentration is used up in the body as a result of physical exercise,

$\boldsymbol{r}$: The rate at which epinephrine is released for energy as a result of physical exercise.

Variables of the Existing Model

$\boldsymbol{g(t)}$: Excess glucose concentration in the blood at time t,

$\boldsymbol{h(t)}$: Excess insulin concentration in the blood at time t,

$\boldsymbol{e(t)}$: Excess epinephrine concentration in the blood at time t,

Assumptions of the Existing Model

i. An increase in glucose in the blood stimulates tissue uptake of glucose and glycogen storage in the liver,
ii. An increase in insulin facilitates the uptake of glucose in tissues and the liver,
iii. An increase in blood glucose results in the release of insulin,
iv. An increase in insulin only resultsin increased metabolism of excess insulin,
v. An increase in epinephrine stimulates tissue uptake of glucose,

vi. An increase in epinephrine facilitates the utilization of insulin,
vii. Physical exercise results in the burning of calories, which facilitates the utilization of glucose by muscles and liver,
viii. Physical exercise results in the burning of calories, which increases the utilization of insulin,
ix. Physical exercise results in the utilization of glucose, which facilitates the release of plasma epinephrine concentration.

The Existing Model Equations

Following from the parameters, variables and assumptions, the existing model by Kwaghkor and Luga (2016) is given as follows

$$\frac{dg}{dt} = -(a+p)g - bh + fe \quad (1)$$

$$\frac{dh}{dt} = cg - (d+q)h + ke \quad (2)$$

$$\frac{de}{dt} = -lg - mh + (n+r)e \quad (3)$$

The Extended Model

The Model equations (1)–(3) are extended by including dietary (diet restriction) as another diabetes control measure under the following additional parameters and assumptions.

Parameters of the Extended Model

S: Rate at which glucose is used up in the blood due to dietary
T: Rate at which insulin is used up in the blood due to dietary
U: Rate at which epinephrine is released due to dietary

Assumptions of the Extended Model

i. Dietary facilitates the use of glucose in the blood
ii. Dietary facilitates the use of insulin in the blood
iii. Dietary facilitates the release of epinephrine in the blood
iv. The rate at which dietary affects glucose, insulin and epinephrine concentration in the blood is equal

The Extended Model Equations

Based on the assumptions and parameters introduced, the new model equations are

$$\frac{dg}{dt} = -(a+p+S)g - bh + fe, \quad (4)$$

$$\frac{dh}{dt} = cg - (d+q+T)h + ke \quad (5)$$

$$\frac{de}{dt} = -lg - mh + (n+r+U)e \quad (6)$$

Stability of the Extended Model

Table 1: Values of the Model Parameters

Parameter/Variable	Value	Source
a	2.9200	Paolo et al. (2002)
b	4.3400	,,
c	0.2080	,,
d	0.7800	,,
f	1.2400	,,
k	0.1400	,,
l	2.9400	,,
m	0.9800	,,
n	0.5300	,,
p	0.0475 Calorie	Hypothetical
q	0.0760 Calorie	,,
r	0.0025 Calorie	,,
S	0.2500 Calorie	Mayoclinic (2019)
T	0.2500 Calorie	,,
U	0.2500 Calorie	,,
$g(0)$	18.3000 mmol/l	Hypothetical
$h(0)$	6.3000 mmol/l	,,
$e(0)$	30.9093 mmol/l	,,

Using the parameter values of Table 1, the model equations (4) – (6) is given as

$$\frac{dg}{dt} = -3.2175g - 4.34h + 1.24e \quad (7)$$

$$\frac{dh}{dt} = 0.208g - 1.106h + 0.14e \quad (8)$$

$$\frac{de}{dt} = -2.94g - 0.98h + 0.6325e \quad (9)$$

which in matrix form, is presented by equation (10) below

$$\begin{pmatrix} \frac{dg}{dt} \\ \frac{dh}{dt} \\ \frac{de}{dt} \end{pmatrix} = \begin{pmatrix} -3.2175 & -4.34 & 1.24 \\ 0.208 & -1.106 & 0.14 \\ -2.94 & -0.98 & 0.6325 \end{pmatrix} \begin{pmatrix} g \\ h \\ e \end{pmatrix} \quad (10)$$

with equation (11) as the characteristic equation given below

$$\left| \begin{pmatrix} -3.2175 & -4.34 & 1.24 \\ 0.208 & -1.106 & 0.14 \\ -2.94 & -0.98 & 0.6325 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \right| = 0 \quad (11)$$

From equation (11), the eigenvalues are given as

$$\lambda_1 = -1.8346 + 1.4366i$$

$$\lambda_2 = -1.8346 - 1.4366i$$

$$\lambda_3 = -0.0218 + 0.0000i$$

Since the eigenvalues are both complex with negative real parts, the system of equations (4) – (6) is said to be uniformly asymptotically stable (Deo and Reghavendra, 1980).

RESULTS AND DISCUSSION

Results

The null – clines of the model equations (1) – (3) and (4) – (6) were simulated to study the effect of physical exercise and the combined effect of physical exercise and dietary (diet restriction) respectively in the management of diabetes. The null – clines of the model equations (1) – (3) and (4) – (6) are presented in equations (12) – (14) and (15) – (17) respectively below

$$g(p) = \frac{fe - bh}{a + p} \quad (12)$$

$$h(q) = \frac{ke + cg}{d + q} \quad (13)$$

$$e(r) = \frac{mh + lg}{n + r} \quad (14)$$

and

$$g(x) = \frac{fe - bh}{a + x}, \quad where\ x = p + S, \quad (15)$$

$$h(y) = \frac{ke + cg}{d + y}, \quad where\ y = q + T, \quad (16)$$

$$e(z) = \frac{mh + lg}{n + z}, \quad where\ z = r + U, \quad (17)$$

A computer program is written in MATLAB to simulate equations (12) – (14) and (15) – (17). The graphs generated from the simulation are presented in Figure 1 and 2 below respectively.

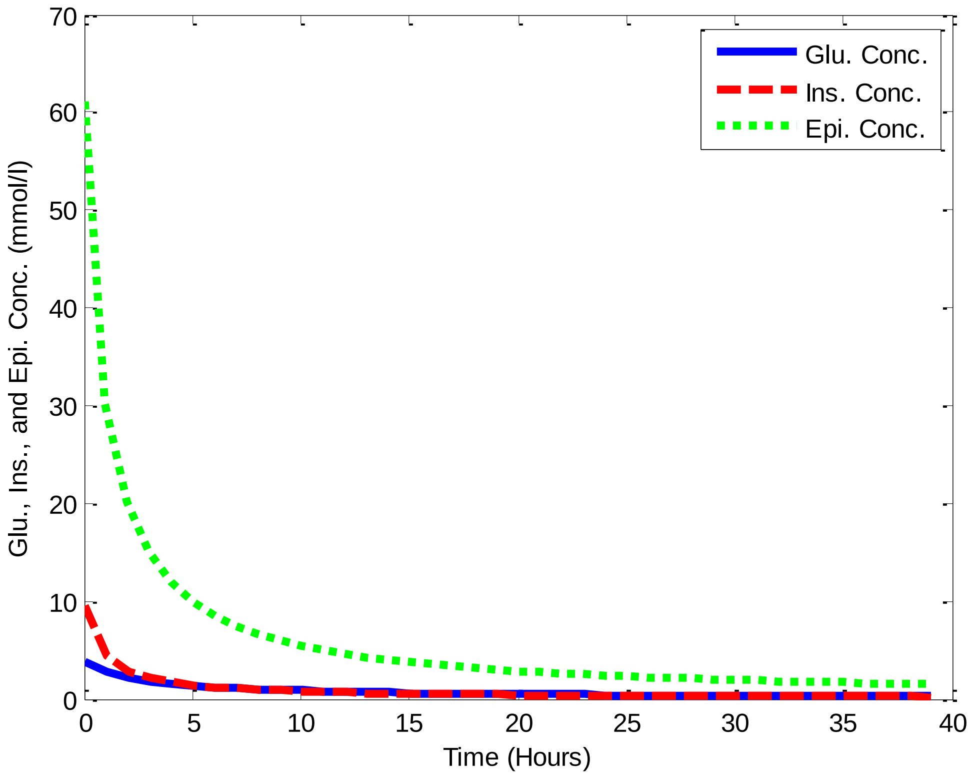

Figure 1: Effect of Physical Exercise on Glucose (Glu.), Insulin (Ins.), and Epinephrine (Epi.) Concentrations (Conc.) in the blood

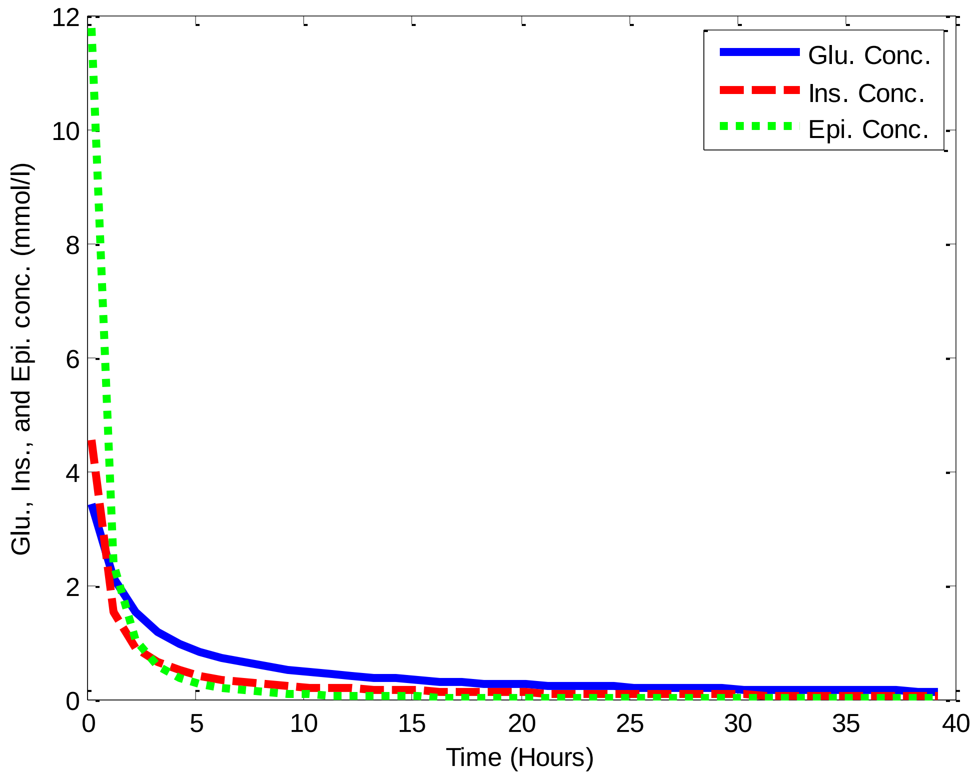

Figure 2: Combined Effects of Physical exercise and dietary (diet restriction) on Glucose (Glu.), Insulin (Ins.), and Epinephrine (Epi.) Concentrations (Conc.) in the blood

Discussion

Figure 1 above is describing the effect of physical exercise (amount of calories burnt) on glucose, insulin and epinephrine concentrations in the blood. It shows that as physical exercise (amount of calories burnt) is maintained, the excessglucose, insulin and epinephrine concentrations returns to their normal level with time.

Figure 2 above is describing the combined effect of physical exercise (amount of calories burnt) and dietary (diet restriction) on glucose, insulin and epinephrine concentrations in the blood. It shows that as physical exercise (amount of calories burnt) and dietary are maintained, the excessglucose, insulin and epinephrine concentrations returns to their normal level with time. Comparing the graph of Figure 2 to that of Figure 1, it is noticed that the excessglucose, insulin and epinephrine concentrations returns to their normal level in Figure 2 with time faster than that of Figure 1. This is to say that in the management of diabetes, physical exercise and dietary as recommended by the physician if combined will reduces plasma glucose to normal levels, improves insulin sensitivity by decreasing the amount of insulin required to improve glucose uptake and also brings the excess epinephrine concentration to normal levels. These results agree with that of Adamu et al (2014).

CONCLUSION

This research work is set out to extend an earlier model for the detection and control of diabetes in the blood. The extension was done by adding to the existing parameters dietary in order to study the combined effects of physical exercise and dietary in the management of diabetes. The extended model equations where found to be uniformly asymptotically stable. The result shows that when physical exercise is combined with recommended dietary in the management of diabetes, the excess glucose, insulin and epinephrine concentrations returns to their normal level faster with time compared to when only physical exercise is used as a control measure. Hence, it is recommended that in the management of diabetes, both physical exercise and dietary should be used as control measures.

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