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THE SOLUTION OF A MATHEMATICAL MODEL FOR COVID-19 TRANSMISSION AND VACCINATION IN NIGERIA BY USING A DIFFERENTIAL TRANSFORMATION METHOD

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ABSTRACT

In this work, Differential Transform Method (DTM) was employed to obtain the series solution of the SIRV COVID-19 model in Nigeria. The validity of the DTM in solving the model was validated by Maple 21's Classical fourth-order Runge-Kutta method. The comparison between DTM and Runge- Kutta (RK4) solutions was performed and there was a good correlation between the results obtained by the two methods. The result validates the accuracy and efficiency of the DTM to solve the model.

Keywords: SIRV, DTM, Runge Kutta, Covid-19, Maple, Nigeria

INTRODUCTION

The novel coronavirus, (SARS-CoV-2) Pandemic has negatively affected all spheres of human lives globally resulting in an economic recession, unemployment, abject poverty, poor quality of life, health system degradation, educational system retro-gradation, and lots more. (Hassan, et. al, 2020; World Bank, 2020). Nigeria like other countries of the world has had its share of the impact of the pandemic since the inception of the first index case reported on February 27, 2020 (NCDC, 2020). The Federal Government of Nigeria and the Nigeria Centre for Disease Control (NCDC) have designed and implemented various measures to limit the spread of the virus by creating public awareness through vital communication strategies using electronic, social, and print media to alert Nigerians on the spread of the virus, closure of all educational institutions, closure of both domestic and international airports, closure of Borders and sea ports, Interstate lockdown, stay-at-home order, restriction in public gatherings, social distancing and the use of face mask (Nnama-Okechukwu, et. al, 2020). The Nigerian's failure to adhere to all measures to prevent transmission of the disease has prompted Nigeria's government to initiate the move to vaccinate her citizens against the SARS-CoV-2. Nigeria's COVID-19 vaccination began on 5th May 2021 and as of 28th March 2021, a total of 932,623 doses of the AstraZeneca/Oxford vaccine have been administered (Okoroiwu, et. al, 2021). To understand the effect of vaccination on the transmission dynamics of the spread of disease, different studies have shown the importance of mathematical approaches in understanding Covid-19 disease dynamics and evaluating the effectiveness of vaccination (Phaijoo and Gurung, 2010). The use of mathematical models to study the spread of infectious diseases has become a very important tool for decision-making policy in public health. Most epidemiological models are represented using systems of non-linear ordinary differential equations. Due to the non-linearity of most of the mathematical models in solving epidemic problems, diverse numerical methods which include approximate, exact, and purely numerical are used repeatedly to find the solutions of the differential equations (Christopher et. al, 2020). However, most of these methods are computationally intensive or require symbolic computations (Oyedepo, et. al, 2018). Some of these numerical methods are Runge Kutta Method, Homotropy Perturbation Method (HPM), Homotropy Analysis Method (HAM), Variational Iteration Method (VIM), and Adomian Decomposition method (Akogwu and Fatoba, 2022), He, 2008; Abbasbandy and Shivanian, 1999; Wazwaz, (2005).

The Concept of DTM was first introduced and applied to solve linear and nonlinear initial value problems in electric circuit analysis (Zhou, 1986). This method which was derived from Taylor's series expansion has been used to solve problems in Mathematics and Physics such as fractional differential-algebraic equations, fourth-order parabolic partial differential equations, fractional-order integrodifferential equations, differential equations, and problems in epidemic models (Nazari and Shahmorad, 2010; Mirzaee, 2011). Differential transform method has also been applied to solve different epidemic models (Attah, et. al, 2022; Adebisi, et.al, 2019; Abioye, et. al, 2018; Christopher et. al, 2020; Eguda, et. Al, 2019; Lawal, et. al, 2018; Ndii, et. al, 2018; Olasebikan, et. al,2021; Omoloye, et. al,2021; Pakwan, et. al, 2021; Peter, et. al, 2012 and 2018).

This work aims to apply the Differential Transformation Method (DTM) to find the approximate series solution for the SIRV COVID-19 model as proposed by (Schlickeiser and Kröger, 2021) and to verify the validity of the method in solving the model using Maple 21's classical fourth-order Runge-Kutta method as a basis for comparison.

MATERIAL AND METHODS

SIRV COVID-19 Model

The Susceptible, Infection, Recovery, Death, and Vaccinated (SIRV) model, is a modification made to the SIR compartmental model and is formulated using a nonlinear first-order differential equation. This model divides the entire population into four compartments, the Susceptible, the Infected, the Vaccinated, and the Recovery compartments. The SIRV model accounts for the vaccination of the Susceptible Population with the vaccination rate.

$$\frac{dS}{dt} = -\beta(t)SI - \sigma(t)S \quad (1)$$

$$\frac{dI}{dt} = \beta S(t)I(t) - \gamma(t)I \quad (2)$$

$$\frac{dR}{dt} = \gamma(t)I \quad (3)$$

$$\frac{dV}{dt} = \sigma(t)S \quad (4)$$

The initial conditions and parameters used are cumulative valves for the various classes extracted from (Okoroiwu, et. al, 2021)**.** The initial conditions and parameters are described in the table below.

Table 1: The values and parameters used in the model are from February 29th, 2020 to March 28th, 2021.

Symbol	Descriptions	Sources
S	Individuals who are susceptible to the disease	1778105 (Okoroiwu1, et. al, 2021)
I	Individuals per unit of time who are infected with the disease	162593(Okoroiwu1, et. al, 2021)
β	Infected rate per unit of time	5.59458259X10^(-08) (Estimated)
γ	Recovery rate per unit of time	0.9244432 (Estimated)
R	Individuals per unit of time who recovered from the disease	150308(Okoroiwu1, et. al, 2021)
σ	Vaccination rate	0.43690 (Estimated)

Definition of Differential Transformation Method

The differential transformation method is a semi-analytical method of solving linear and nonlinear systems of ordinary differential equations (ODE) to obtain approximate series solutions. DTM has an edge over other techniques due to its ability to provide the desired accuracy and reliable series solution with a remarkable convergence rate requires less computational space and yields high accuracy and does not require linearization and perturbation parameters (Pakwan, et. al, 2021).

Derivation of DTM

The differential transformation $F(k)$ of a function $f(t)$ is defined as follows:

$$F(k) = \frac{1}{k!}\left[\frac{d^k f(t)}{dt^k}\right]_{t=0} \qquad (5)$$

where $f(t)$ is the original function and $F(k)$ is the transformedfunction.

The differential inverse transformation of $F(k)$is defined as follows:

$$f(t) = \sum_{k=0}^{\infty} t^k F(k) \qquad (6)$$

From (5)and (6), respectively, the arbitrary function $f(t)$is expanded in the Taylor series around a point $t = 0$is defined as;

$$f(t) = \frac{t^k}{k!}\sum_{k=0}^{\infty}\left[\frac{d^k f(t)}{dt^k}\right]_{t=0} \qquad (7)$$

If $g(t)$ and $h(t)$are two uncorrelated functions with t, where $G(t)$) and $H(t)$are the transformed functions corresponding to $g(t)$ and $h(t)$) then, the fundamental mathematical operationsperformed by differential transform are listed blow

Table 2: **The mathematical operations of the DTM**

S/N	Original function	Transformed Function
1	$F(k) = G(t) \pm H(t)$	$f(t) = g(t) \pm h(t)$
2	$F(k) = aG(k)$, where a is a constant	$f(t) = aG(t)$
3	$F(k) = (k+1)F(k+1)$	$f(t) = \frac{dg(t)}{dt}$
4	$F(k) = (k+1)(k+2) + \cdots (k+m)F(k+m)$	$f(t) = \frac{d^k g(t)}{dt^k}$
5	$F(k) = \delta(k)$, where δ is the Kronecker delta	$f(t) = 1$
6	$F(k) = \delta(k-1)$	$f(t) = t$
7	$F(k) = \delta(k-m) = \begin{cases} 1, k^{\ m} \\ 0, k_{\neq m} \end{cases}$	$f(t) = t^m$
8	$F(k) = \sum_{m=0}^{k} G(m)H(k-m)$	$f(t) = g(t)h(t)$
9	$F(t) = e^{(\lambda t)}$	$f(t) = \frac{\lambda^k}{k!}$
10	$F(t) = (1+t)^m$	$f(t) = \frac{(m(m-1)\ldots(m-k+1))}{k!}$

Employing the differential transformation method and using the mathematical operational properties 1, 2, 3, and 8 from table 2 to solve our SIRV COVID-19 model, we have

$$S(k+1) = \frac{1}{k+1}\left(-\beta\sum_{l=0}^{k} S(l)\,I(k-l) - \sigma S(l)\right) \qquad (8)$$

$$I(k+1) = \frac{1}{k+1}\left(\beta\sum_{l=0}^{k} S(l)\,I(k-l) - \gamma I(l)\right) \qquad (9)$$

$$R(k+1) = \frac{1}{k+1}\left(\gamma I(l)\right) \qquad (10)$$

$$V(k+1) = \frac{1}{k+1}\left(\sigma S(l)\right) \qquad (11)$$

SIMULATION RESULTS AND DISCUSSIONS

The initial conditions with the values of the parameters for the model in table1 are used for the numerical simulation and it is implemented with the help of Maple21 programming software. Up to 5[th] terms approximations to the solutions of $S(t)$, $I(t)$, $R(t)$ and $V(t)$ in a closed form are determined. The closed form of the solution where k = 0, 1, 2, 3, 4 and 5 can be written as;

$$s(t) = \sum_{n=0}^{k} S(k)t^k = 1778105 - 793028.4282t + 173868.3946t^2 - 25612.29107t^3 + 2823.011196t^4 - 249.0591081t^5$$

$$i(t) = \sum_{n=0}^{k} I(k)t^k = \\ = 162593 - 134133.1192\, t + +61367.67585\, t^2 - -18618.98755t^3 + 4277.525509t^4 - 788.4787467t^5$$

$$r(t) = \sum_{n=0}^{k} R(k)t^k = 150308 + 150307.4729t - 61999.01036t^2 + 18910.24476t^3 - 4303.034215t^4 + 790.8631366t^5$$

$$v(t) = \sum_{n=0}^{k} V(k)t^k = 923623 + 776854.0745t - 173237.0601t^2 + 25321.03386t^3 - 2797.502492t^4 + 246.6747183t^5$$

Graphical Presentation of The Simulation Result

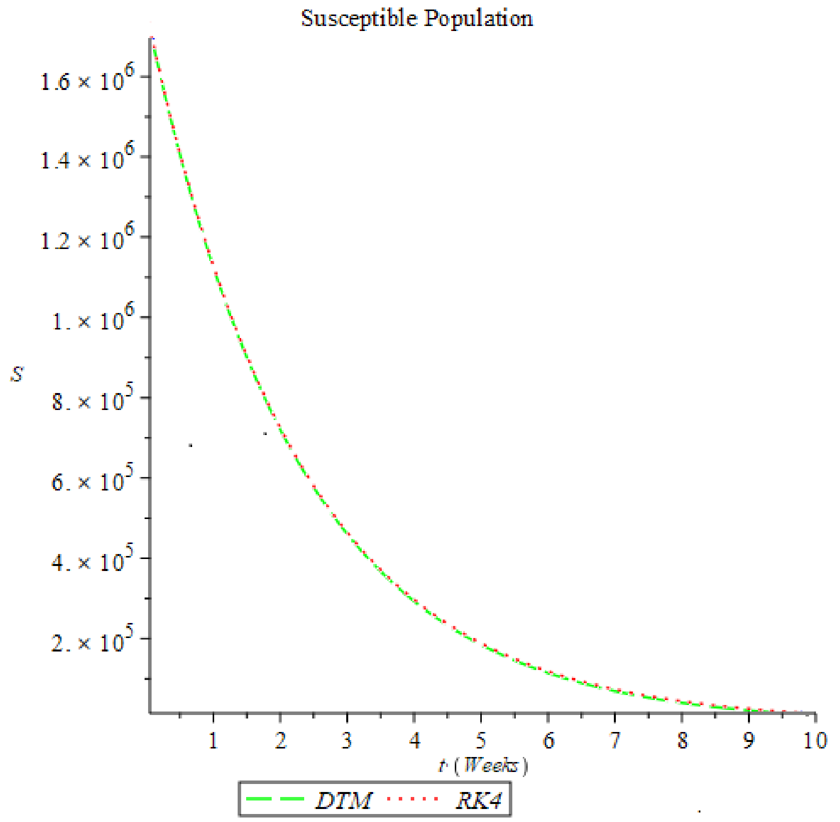

Figure 1: Plot showing the solution of the susceptible population by DTM and RK4

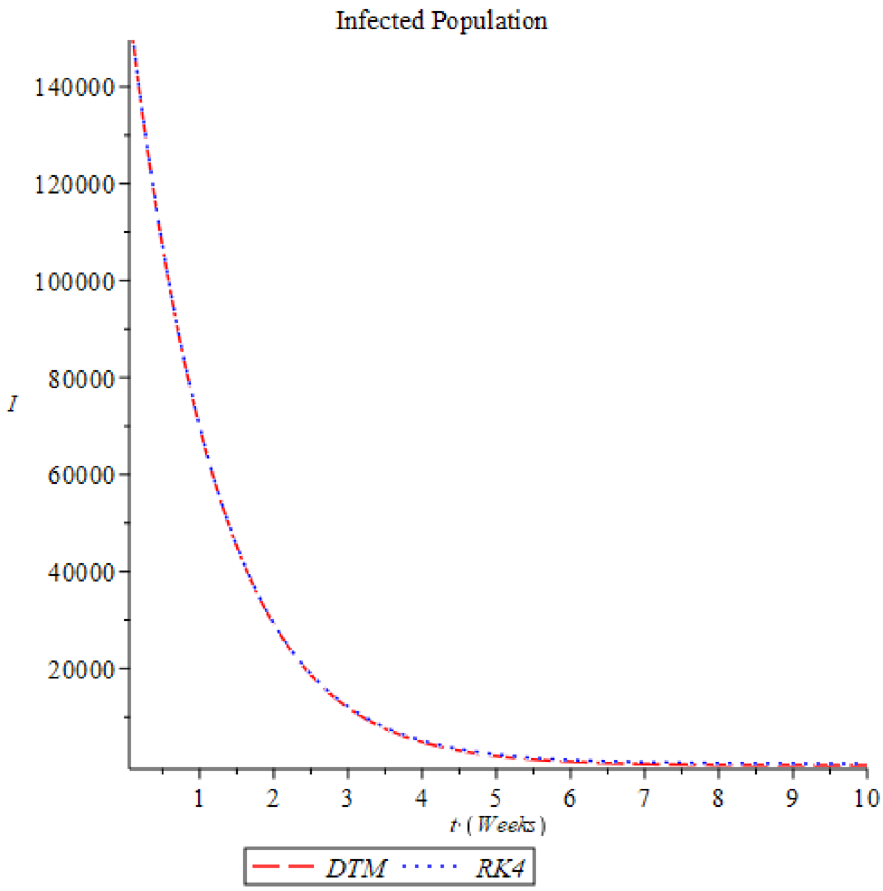

Figure 2: Plot showing the solution of the infected population by DTM and RK4

The graph in Figures 1 and 2 shows that the impact of vaccination on the susceptible populationhas significantly reduced the spread of the disease, thereby leading to a drastically deceased in the susceptible and the infected population. Also, the two methods show a good correlation.

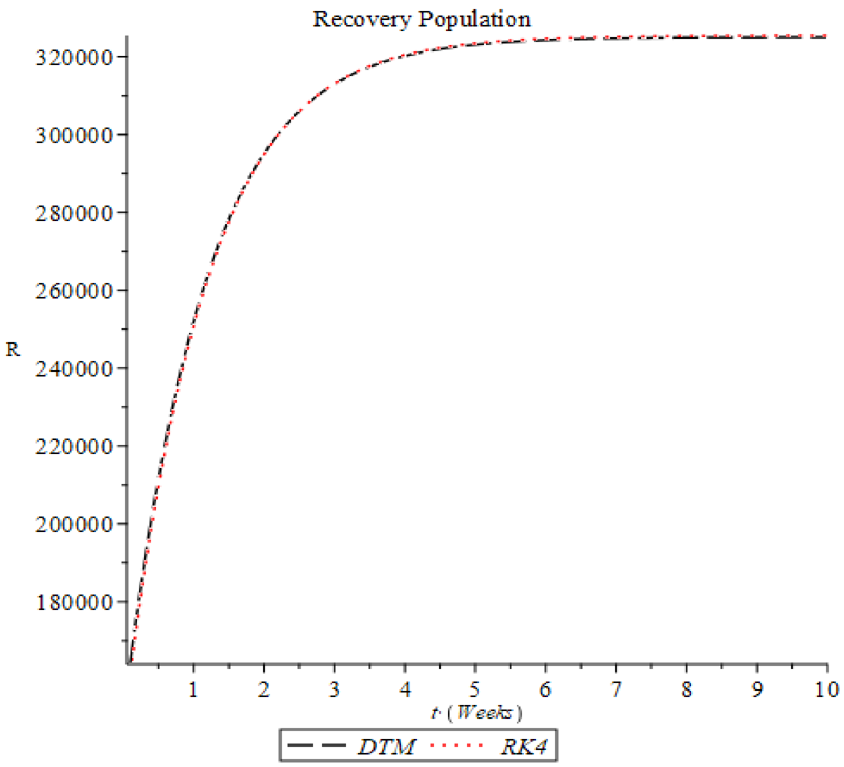

Figure 3: Plot showing the solution of recovery population by DTM and RK4

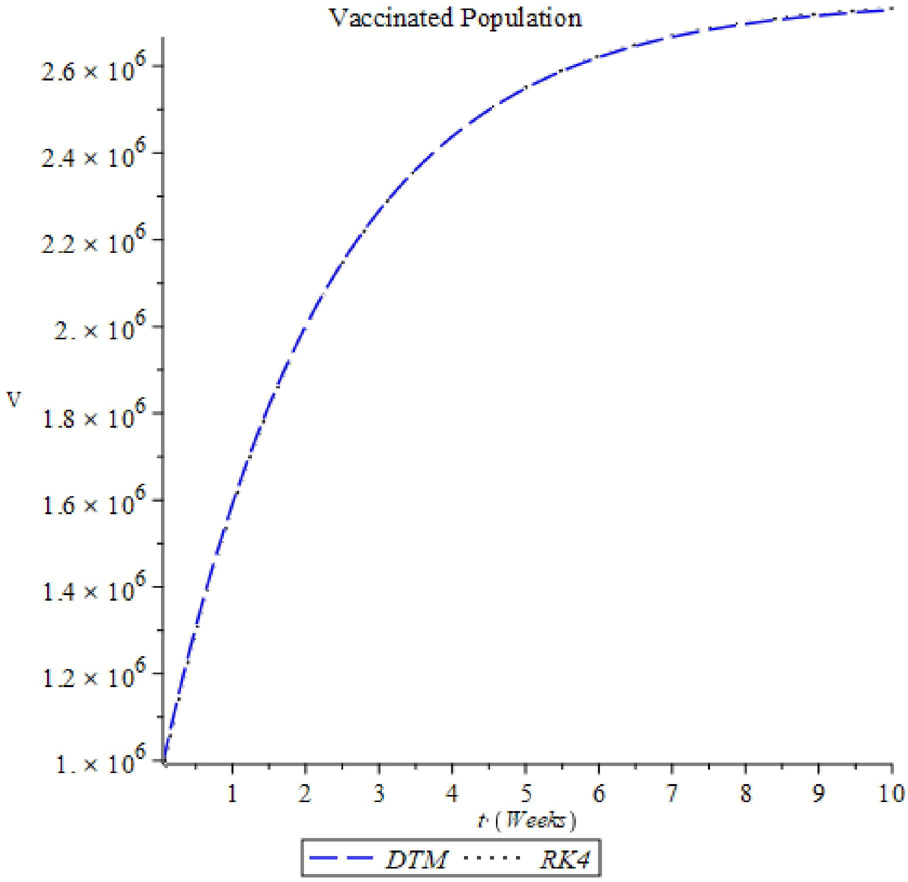

Figure 4: Plot showing the solution of the vaccinated population by DTM and RK4

It was observed from Figures 3 and 4, that the two numerical methods show the same graphical result. Also, as the vaccination of the susceptible population increases, the recovery population increases and tends toward linearity as a result of the migration from the infected population.

CONCLUSION

In this paper, the DTM was used to solve the series solution of the SIRV COVID-19 disease model. The accumulated result of the Susceptible Class, Infected Class, Recovery Class, and Vaccination Class as of March 28th, 2021 was used as the initial conditions. The Simulation result was obtained with the Maple 21 programming software, the efficiency and the accuracy of the DTM were validated with Maple 21's in-built classical fourth-order Runge-Kutta method (RK4), and the graphs from the two numerical methods show a good correlation. It was concluded that the mathematical modeling of epidemiology can be solved using the differential transformation method because of it is efficiency and reliability. Also, it was observed from this research that the effect of vaccination on COVID-19 disease transmission has drastically reduced as well as the rate of infection, leading to an exponential increase in the recovery population. This shows that the COVID-19 disease is tending toward extinction with effective vaccination in Nigeria.

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