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QUEUEING THEORY IN SOLVING TELE-TRAFFIC PROBLEM

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ABSTRACT

Information technology has made the world a global village and tele communication plays an indispensible role in this. Queues are predominant features in information processing and transfer. This work examines queue model in relation to telecommunication transfer/ processing problems. Some of the prevalent questions in tele-traffic are "how do we minimize time wastage (how long do I wait?) in the system and what will be the effect on waiting time if the number of operations are altered. In this work, tele-traffic problem was formulated and solution established through a model. Focasting ideas were also suggested. The queue theory has been applied to answer the above questions. The model has shown increased service satisfaction and in turn minimizes cost.

Keywords: Queueing theory, tele-traffic, information processing

INTRODUCTION

A queuing process is characterized by the arrival of units which require service at one or more service facilities. The units demanding service may be people, machines or other elements and all are called customers. A service may be performed by moving the customer to the server or vice versa. The principal actors in a queuing situation are the customers and the servers. Customers are generated from a source and on arrival at service facility they can start service immediately or wait in a queue if the facility is busy (Harris 1997). In most queuing processes either some customers have to wait in order to obtain service or the servers are idle waiting for customers to arrive. Both types of waiting are costly hence the need to strike a balance between both types of costs (Dar drecht 1992).

According to Zhiqlag et al (2015),the problem that arises when a telephone subscriber attempts to make a call could be "what is the average waiting time and what is the chance that the subscriber obtains service immediately without waiting?" what is the effect on waiting time if the number of operations is altered? e.t.c. The aim of the system is to reduce waiting time to the minimum thereby increasing service satisfaction.

In 1908 a Dannish mathematician Erlang, A. K. studied the problem of congestion of the Copenhagen telephone company and His work provided a solution that brought a balance between reduced waiting time and improved facility.(Dash and Kajiji 1998)

Queue theory are waiting lines and queue theory is the study of waiting line processes through the use of mathematical and simulation models (Trederich and Gerald 2008). The basic queue contexts are; units from some service arrive at a service facility in a queue or system of queues; wait if necessary, receive service at a time(s) determined by some service policy/discipline and depart after service has been completed. The statistical pattern by which units arrive at the service facility is the "arrival time distribution". According to Chikwendu and Okoye (2016) the arrival process is called random arrival or Poisson arrival. The number of arrivals in a fixed period of time has the Poisson distribution. The rule by which units are selected from the queue structure and service is the service discipline viz. First come first serve (FCFS), first in last out (FILO), first in first out (FIFO) e.t.c. (Taha 2006). The service facility consists of one or more service channels in parallel with each channel having one or more servers in series. The time required to service completely a unit for any server is the server's service time or holding time.

Kobayashi and Konhen (1997) stated the two major types of service facility as the single and multiple facility.

Asmissen (1987) saw queue model as a branch of probability theory which studies mathematical models of various kinds of real queues. These models are service processes; for example, In an automatic telephone exchange one of the basic characteristics is the proportion p of calls lost i.e. the limit of p as $t \to \infty$ (if it exists) of the ratio r(t)/e(t) of the number r(t) of calls lost up to time t to the number e(t) of calls which arrive up to the same time.

The Poisson distribution results from the occurrences that can be described by a discrete random variable of the system (Hamburg 1991). The probability of exactly x occurrences in the Poisson distribution is

$$F(k) = \frac{\mu^x e^{-\mu}}{x!} \quad \text{for } x= 0,1,2,\ldots \qquad (1)$$

where μ is the mean number of occurrences per arrival and is the natural logarithm e. The Poisson process has exponential inter-arrival times with parameter μ.

This work will consider only the M/M/1 queue model (where M=arrival pattern, M= departure pattern) with the following form;

i) Mean time between arrivals is $1/\mu$.
ii) Mean time between departure is $1/\gamma$.
iii) Probability that n – customers will arrive at the system during service time T is p (n arrivals in time interval T) is

$$\frac{\mu T^n e^{-\mu T}}{n!} \qquad (2)$$

MATERIALS AND METHODS

In this work we express the number of customers in the system of time (t) in terms of μ and γ

We make the following assumptions;

i. That Δt is so small that $\Delta t^2 \to 0$ as $n \to \infty$
ii. During Δt, not more than one customer can arrive and not more than one customer can be served.

The basic relations for the M/M/1 queue model are as follows;

Let there be zero customers in the system of time t i.e. the probability of no customer in the system at t + Δt is

$$P_0(t+ \Delta t) = p_0(t) + (1- \mu\Delta t)(1) \qquad (3)$$

Since if there is no customers in the system, the probability of no service completion /departure is 1.

Now suppose that there is no customers in the system at time t i.e.

Time t+ Δt time t $\mu\Delta t$
1 n=0 1- $\mu\Delta t - \gamma\Delta t$ (4)
n=1 $\gamma\Delta t$

We now wish to find the probability of no customer in the system at t+ Δt

$\therefore P_0$ (t+ Δt) = P_0 (t+ Δt) + P(t(1- $\mu\Delta t(\mu\Delta t)$ (5)

From (1) and (3), the overall probability having no customer in the system at time t+ Δt is

P_{n+1} (t+ Δt)= P_n (t)(1-Δt) +P_1(t)(1- $\mu\Delta t$)($\gamma\Delta t$)
= P_{n+1} (t) – P_n (t) $\mu\Delta t$ + P_1(t)(1- $\mu\Delta t$) – P_1(t) $\mu\gamma(\Delta t)$ (6)
= P_1 (t) – P_0 (t)$\mu\Delta t$ + P_1(t)$\gamma\Delta t$) as $(\Delta t)^2 \to 0$

Substituting P_0 (t) for P_0 (t+ Δt) in (4) gives

P_1 (t) = P_0(t) $\mu\gamma$ (7)

put n = 1 in (4) above to get

P_2 (t) = P_1(t) = P_1(t)$\frac{\mu+\gamma}{\mu}$ - P_0(t)$\frac{\mu}{\gamma}$ (8)

= P_0(t)($\frac{\mu}{\gamma}$)($\frac{\mu+\gamma}{\mu}$) - P_0(t)($\frac{\mu}{\gamma}$)

= P_0(t)($\frac{\mu}{\gamma}$) $\frac{\mu+\gamma-1}{\mu}$ = $\frac{\mu^2}{\gamma^2}$$P_0$(t) (9)

Use (4) in (5) and let n = 1 to get,

P_2(t) = P_1(t)$\frac{\mu+\gamma}{\mu}$ - P_0(t)($\frac{\mu}{\gamma}$) (10)

= P_0(t)($\frac{\mu}{\gamma}$) $\frac{\mu+\gamma-1}{\mu}$ = P_0(t)[($\frac{\mu}{\gamma}$) $\frac{\mu+\gamma}{\mu}$ - $\frac{\mu}{\gamma}$]

= [$\frac{\mu(\gamma+\mu)-\mu\gamma}{\mu}$]$P_0$(t) = ($\frac{\mu}{\gamma}$)^2 P_0(t) (11)

From (5) and (8) it is obvious that

P_1(t) = P_0(t)($\frac{\mu}{\gamma}$)

P_2(t) = P_0(t)($\frac{\mu}{\gamma}$)^2

Hence P_n(t) = P_0(t)($\frac{\mu}{\gamma}$)^n

But $\sum_{n=0}^{\infty} P_n = 1$

It implies that $\sum_{n=0}^{\infty}(\frac{\mu}{\gamma})^n P_0 = 1$

$\therefore P_0 = \frac{1}{\sum_{n=0}^{\infty}(\frac{\mu}{\gamma})^n}$ = 1/ ($\frac{\mu}{\gamma}$)+($\frac{\mu}{\gamma}$)^3 + ($\frac{\mu}{\gamma}$)^2+($\frac{\mu}{\gamma}$)^3+… (12)

(10) converges to 1/(1- $\frac{\mu}{\gamma}$) = ($\frac{1-\mu}{\gamma}$)

$\therefore$ P_n = ($\frac{\mu}{\gamma}$)^n ($\frac{1-\mu}{\gamma}$) (13)

Now the expected number of customers in the system L is found as follows;

L = $\sum_{n=0}^{\infty} nP_n$ = $\sum_{n=0}^{\infty} nP_{n(\frac{\mu}{\gamma})^n}$

= $\sum \quad n(1-(\frac{\mu}{\gamma})(\frac{\mu}{\gamma})^n$ (14)

Using (10) in (11) gives

1- ($\frac{\mu}{\gamma}$) $\sum_{n=0}^{\infty} nP_{n(\frac{\mu}{\gamma})^n}$ = (1- ($\frac{\mu}{\gamma}$))(0(($\frac{\mu}{\gamma}$)^0+ 1($\frac{\mu}{\gamma}$)^1+ 2($\frac{\mu}{\gamma}$)^2+…)

= (1- ($\frac{\mu}{\gamma}$))[0 + 1($\frac{\mu}{\gamma}$)^1+ 2($\frac{\mu}{\gamma}$)^2+…] (15)

(13) is a convergent series with sum [$\frac{\frac{\mu}{\gamma}}{(\frac{1-\mu}{\gamma})^2}$]

Hence L = $\frac{(1-\frac{\mu}{\gamma})\frac{\mu}{\gamma}}{(\frac{1-\mu}{\gamma})^2}$ = $\frac{\frac{\mu}{\gamma}}{(1-\frac{\mu}{\gamma})}$ = $\frac{\mu}{\gamma-\mu}$ (16)

And the average time spent in the system W is obtained thus;

W = $\frac{\text{expected number in the system}}{\text{arrival rate}} = \frac{L}{\mu}$ (Brandwuyn and Yow 1998)

$\therefore$ $\frac{\mu}{\mu(\gamma-\mu)}$ = $\frac{1}{\gamma-\mu}$ from (16)

$\therefore$ W =$\frac{1}{\gamma-\mu}$ (17)

W_q = [$\frac{1}{\gamma-\mu}$] - $\frac{1}{\gamma}$ = $\frac{\gamma-(\gamma-\mu)}{\gamma(\gamma-\mu)}$

= $\frac{\mu}{\gamma(\gamma-\mu)}$ (18)

Cidon et al (2022) stated the mean number in the queue L_q as; Mean number in the system – Mean number in service.

= $\frac{\mu}{(\gamma-\mu)}$ - $\frac{\mu}{\gamma}$ = $\frac{\mu\gamma-\mu(\gamma-\mu)}{\gamma(\gamma-\mu)}$

$\therefore$ Lq = $\frac{\mu^2}{\gamma(\gamma-\mu)}$ (19)

RESULTS AND DISCUSSION

The Tele-Traffic Problem

Consider the problem of the analysis of queue delay in one type of cross bar exchange that is used to determine exchange by exchange when each requires to be extended to meet at the appropriate grades of service , the forecast demand at the end of the period. Consider an exchange system with two originating group selection units channel traffic out to other exchanges. These units channel traffic to the line selection unit to which the called subscriber is connected. In this system the incoming selection units are largely concerned with routing of incoming calls from the exchange (Pankop et al 2017).

The control equipment in exchange carries out the following functions;

i. Recognition of the subscriber's intention of making a call signaled by his handset and to know if the call subscriber is busy. This is done to exchange by line circuits, one circuit for each subscriber connected and;
ii. Setting up a call and monitoring the calling/called line. It finally clears the connection at the end of the calls.

In carrying out these functions delays may occur at any device or all of the line unit device and this gives rise to an overall dial tone delay. The problem then is "how would this delay be minimized? How best could this exchange system be managed to optimize service ?"

Queue processes and Tele-Traffic Model

According to Afolalu et al (2019) Telecommunication mechanisms have been conceptualized simply as links between two points to send signals. Telecommunication networks are at present characterized by clusters and nodes, interconnecting nodes where information is processed and correctly addressed to the output links. A close examination of the queue process reveals that the evolving queue from the pre-selection and group line selection evolve a nested sequence.

In the system switches are operated in the primary selection which is seized to prevent any attempt to have two calls switched on simultaneously through one primary selection. The information path is used for information transfer between coupler and marker/marking relays. The selection coupler is held for some time thus may have to wait for the primary selection to become free and held throughout the waiting time. The relationship between the tele-traffic problem and the queuing theory/model is obvious from the queue process and its probable delays at the unit line devices. This accounts for the usefulness of queue models in solving the tele-traffic problems. The task now is to identify and apply the appropriate queue model and to obtain the appropriate estimate of the expected time spent in the system $W_{q.\ i.e.}$

W_q = $\frac{\mu}{\mu(\gamma-\mu)}$ (equation 16 above)

The sequence which occurs has the following features; approximate random arrivals, constant service time or general service time with small coefficient of variation, one or two services, service in random order and a finite number of potential customers (Boroukor 1976). Consequently the following assumptions are adopted;

1) The number of potential customers is infinite.
2) The arrival distribution is exactly Poisson in form, and
3) Arrival at the selection queue is random in an exchange.

The following illustration shows how queue theory is used to solve tele-traffic problem.

Consider a telephone exchange whose inter-arrival time has an exponential distribution and conversion has a normal distribution with mean five and standard deviation one represented in the table below.

Problem 1

Lines	Rate	Mean	S.D.
5	2/min	5	1

When all five lines are busy, the caller simply receives a busy signal. Find the probability that;
(1) exactly three lines are busy. (2) The system is totally occupied. (3) Find the average number of busy servers.

Solution to problem 1

Recall that $P_j = \frac{(\frac{\mu}{\gamma})^j/j!}{(\frac{\mu}{\gamma})^k/k!}$

(1) From the problem,

$$\mu = 2, \quad \frac{1}{\gamma} = 5$$

Hence $\frac{\mu}{\gamma}$ =10 and No of lines (S)=5

P(exactly 3 lines busy) means j = 3

$P_3 = \frac{10^3/3!}{10^k/k!}$

But $\sum_{k=0}^{5} 10^k/k!$ = $(10)^0/1!+10/1+10^2/2!+10^3/3!+10^4/4!+10^5/5!$

= 1477.667.

$P_3 = \frac{10^3/3!}{1477.667} = \frac{166.67}{1477.67}$ = 0.113

Hence an average of 3 lines would be using 11.3% of the time.

(2) P(the system is totally occupied) = set j = s in P_j and s = 5. i.e.

$P_s = \frac{(\frac{\mu}{\gamma})^s/s!}{(\frac{\mu}{\gamma})^k/k!}$

$P_5 = \frac{(\frac{\mu}{\gamma})^5/5!}{(\frac{\mu}{\gamma})^k/k!} = \frac{10^5/5!}{\sum_{k=0}^{5} 10^k/k!} = 833.33 / \sum_{k=0}^{5} 10^k/k!$

But $\sum_{k=0}^{5} 10^k/k! = 1477.67$

P_5 = 833.33/1477.67 = 0.564

i.e. the system is totally occupied 56.4% of the time.

(3) P(average number of busy servers) = N = $\left(\frac{\mu}{\gamma}\right)\left[1 - B\left(S, \frac{\mu}{\gamma}\right)\right]$

= 10[1-B(5,10)]

But B(5,10) = 0.564

P(average number of busy servers) = 10[1- 0.564]

= 10(0.436) = 4.36.

The system will be busy with probability 0.56 and on the average of
over lines will be busy.

Problem 2

Customers arrive at a service point with one cashier according to a Poisson distribution with mean 20 per hour. Service time per customer is exponential with mean 2.5 minutes. Calculate (1) the traffic intensity of the system. (2) the probability that the server is not idle. (3) the expected number of customers in the system. (4) the expected waiting time. (5) the probability that there are four customers in the system.

Solution to problem 2

From the question μ = 20 per hour = 1/3 per minute, $\gamma = \frac{1}{2.5}$

(1) Traffic intensity P = $\frac{\mu}{\gamma} = \frac{1}{3}(2.5) = \frac{5}{6}$

(2) Probability that the server is idle P_0 = 1- p

Probability that server is not idle = 1 - P_0 = P

$$P = \frac{5}{6}$$

(3) The expected number of customers in the system is given by

$$\frac{P}{1-P} = \frac{\frac{5}{6}}{\frac{1}{6}} = 5$$

(4) The expected waiting time in the system is $L_s = \mu$

$$\mu = 5\left(\frac{60}{20}\right) = 15 \text{ minutes}$$

(5) The Probability that there are four customers in the system is

$P_4 = (1 - P)\, P^4$

$(1 - \frac{5}{6})(\frac{5}{6})^4 = \frac{625}{7776}$

From the results, the model has helped to obtain the approximate period the system was busy or idle. It is therefore possible to optimize service satisfaction of the customer in a tele-traffic problem.

CONCLUSION

The telecommunication and traffic system in Nigeria face the problem of congestion. A system that will ease congestion and make for improved services will definitely maximize customers satisfaction (Adeniran et al 2021). Early works in queue theory were highly concerned with practical problems associated with telephone and aimed at increasing customers satisfaction. Queue theory has been used to deal with priority assignments on response time in time-sharing system. This has helped in the production and evaluation of the performance of different computer systems (especially network systems).

The work has examined queue models in relation to tele-traffic system. The knowledge and application of queue theory has helped immensely to improve services thereby increasing customer satisfaction. It also makes management more plausible.

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