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TWO-STEP HYBRID BLOCK BACKWARD DIFFERENTIATION FORMULAE FOR THE SOLUTION OF STIFF ORDINARY DIFFERENTIAL EQUATIONS

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ABSTRACT

In this study, we construct two -Step Second derivative hybrid block backward differentiation formula. The newly proposed scheme was derived based on interpolation and collocation approach. The discrete schemes were obtained from the continuous scheme. The derived method is applied to solve non-linear systems of stiff ordinary differential equations. The numerical result obtained using newly constructed method showed that it is good for stiff systems.

Keywords: Interpolation and Collocation, Backward Differentiation Formula, Hybrid, Stiff ODEs

INTRODUCTION

We shall be concerned with approximating the initial solution $y(x)$ to a problem of the form

$$y' = f(x, y) \text{ for } a \le x \le b$$

with an initial condition $y(a) = \alpha$. (1)

Researchers have developed several methods most of their works on the development of more efficient and accurate methods for the solutions of stiff and non-stiff problems. Fatunla (1988,1991) introduced numerical integrators for stiff and highly oscillatory differential equations and L-stable fourth-order explicit one-step numerical integration formulae which demand no matrix inversion to cover-up effectively with systems of ordinary differential equations with large Lipschitz constants.

The implicit integration procedure proposed by Fatunla was further developed in (1991) to manage a larger class of stiff systems as well as those with extremely oscillator solutions together with a productive computational method. Baiyeri *et al*. (2015) modified extended backward differentiation formula. Some implicit schemes were developed based on the linear multi-step method. Timothy *et al*. (2012) also proposed a new class of one-step hybrid methods for the numerical solutions of ordinary differential equations. The methods were derived by the discreet choice of coefficients for the class of hybrid methods with one off step point. Zurni *et al*. (2016) construct direct Solution of second-order ordinary differential equation using a single-step hybrid block method of order five. The method is developed using interpolation and collocation techniques. Kumleng *et al*. (2013) proposed efficient A-stable numerical methods for the solutions of stiff differential equations. These methods are of uniform order four and six for the three and five step methods respectively. The stability analysis of the two methods indicates that the methods are A–stable, consistent and zero stable. Hojjati (2015) developed a class of multistep methods based on super-future technique for solving initial value problems. The methods were derived using the super future point for improving the region of absolute stability. Umaru et al. (2010) developed Fully Implicit Four Point Block Backward Difference Formulae for Solving First Order Initial Value Problems. The four steps Backward Differentiation Formulae (BDF) were reformulated for applications in the continuous form. Stiff initial value problems are solved using the various stages of the derived modified extended backward differentiation formula with efficient variable order and stage I is the best scheme for solving stiff initial value problem. It converges better than the stage II and stage III with the lowest absolute error value. Hence is the most accurate.

DERIVATION OF THE BLOCK METHOD

Consider power series approximate solution in the form

$$y(x) = \alpha_0 + \alpha_1 x + \cdots + \alpha_p x^p = \sum_{j=0}^{p} \alpha_j x^j \qquad (2)$$

$p = r + s - 1$

Where r and s are the number of interpolation and collocation points respectively.

Which is twice-continuously differential function of $y(x)$.

$$y'(x) = \sum_{j=1}^{r+s-1} j\alpha_j x^{j-1} \quad (3)$$

with the second derivative given by

$$y''(x) = \sum_{j=2}^{r+s-1} j(j-1)\alpha_j x^{j-2} \quad (4)$$

where $x \in [a, b]$, the $a's$are real unknown parameters to be determined.

Collocating (3) at $x = x_{n+1}, x_{n+\frac{3}{2}}, x_{n+2}$ and interpolating (2) at $x = x_n, x_{n+1}$ to give the systems of equation.

$$y(x) = \sum_{j=0}^{6} a_j x^j = y_n \quad (5)$$

$$y'(x) = \sum_{j=1}^{6} j a_j x^{j-1} = f_{n+j} \quad (6)$$

$$y''(x) = \sum_{j=2}^{6} j(j-1) a_j x^{j-2} = f_{n+j} \quad (7)$$

Expressing the system of equations (5) -(7) in the form $XA = Y$ as

$$\begin{bmatrix} 1 & x_n & x_n^2 & x_n^3 & x_n^4 & x_n^5 & x_n^6 \\ 1 & x_{n+1} & x_{n+1}^2 & x_{n+1}^3 & x_{n+1}^4 & x_{n+1}^5 & x_{n+1}^6 \\ 0 & 1 & 2x_{n+1} & 3x_{n+1}^2 & 4x_{n+1}^3 & 5x_{n+1}^3 & 6x_{n+1}^5 \\ 0 & 1 & 2x_{n+3/2} & 3x_{n+3/2}^2 & 4x_{n+3/2}^3 & 5x_{n+3/2}^3 & 6x_{n+3/2}^5 \\ 0 & 1 & 6x_{n+2} & 3x_{n+2}^2 & 4x_{n+2}^3 & 5x_{n+2}^3 & 6x_{n+2}^5 \\ 0 & 0 & 2 & 6x_{n+1} & 12x_{n+1}^2 & 15x_{n+1}^3 & 30x_{n+1}^3 \\ 0 & 0 & 2 & 6x_{n+2} & 12x_{n+2}^2 & 15x_{n+2}^3 & 30x_{n+2}^3 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} = \begin{bmatrix} y_n \\ y_{n+1} \\ f_{n+1} \\ f_{n+\frac{3}{2}} \\ f_{n+2} \\ g_{n+1} \\ g_{n+2} \end{bmatrix} \quad (8)$$

Using Maple software and inverting the matrix in (8) we propose a continuous scheme for $k - step$ of hybrid block extended second derivative backward differentiation formula.

$$y(x) = \sum_{j=0}^{k} \alpha_j(x) y_{n+j} + h\beta_{k-1}(x) f_{n+k-1} + h\beta_{\frac{2k-1}{2}} f_{n+\frac{2k-1}{2}} + h\beta_k(x) f_{n+k} + h^2\gamma_{k-1}(x) g_{n+k-1} + h^2\gamma_k(x) g_{n+k}$$

(9)

The columns of $X^{-1`}$ give the continuous coefficients as:

$$\alpha_0(x) = 1 - \frac{9x}{2h} + \frac{33x^2}{4h^2} - \frac{63x^3}{8h^3} + \frac{33x^4}{8h^4} - \frac{9x^5}{8h^5} + \frac{x^6}{8h^6}$$

$$\alpha_1(x) = \frac{9x}{2h} - \frac{33x^2}{4h^2} + \frac{63x^3}{8h^3} - \frac{33x^4}{8h^4} + \frac{9x^5}{8h^5} - \frac{x^6}{8h^6}$$

$$\beta_1(x) = \frac{69x}{20} - \frac{733x^2}{40h} + \frac{7529x^3}{240h^2} - \frac{1983x^4}{80h^3} + \frac{661x^5}{80h^4} - \frac{263x^6}{240h^5}$$

$$\beta_{\frac{3}{2}}(x) = -\frac{52x}{5} + \frac{202x^2}{5h} - \frac{913x^3}{15h^2} + \frac{221x^4}{5h^3} - \frac{77x^5}{5h^4} + \frac{31x^6}{15h^5}$$

$$\beta_2(x) = \frac{69x}{20} - \frac{553x^2}{40h} + \frac{5189x^3}{240h^2} - \frac{1313x^4}{80h^3} + \frac{481x^5}{80h^4} - \frac{203x^6}{240h^5}$$
$$\gamma_1(x) = \frac{147xh}{40} - \frac{1019x^2}{80} + \frac{8207x^3}{480h} - \frac{1779x^4}{160h^2} + \frac{563x^5}{160h^3} - \frac{209x^6}{480h^4}$$
$$\gamma_1(x) = -\frac{27xh}{40} + \frac{219x^2}{80} - \frac{2087x^3}{480h} + \frac{539x^4}{160h^2} - \frac{203x^5}{160h^3} + \frac{89x^6}{480h^4}$$

Evaluating (9) at x_n,$x_{n+3/2}$andx_{n+2} yields the following discrete methods which constitute the new two-step block method. Therefore, the hybrid block method is;

$$y_n - y_{n+1} = \frac{h}{330}\left[733f_{n+1} - 1616f_{\frac{3}{2}} + 553f_{n+2}\right] + \frac{h^2}{660}[40g_n + 1019g_{n+1} - 219g_{n+2}]$$
$$y_{n+3/2} = -\frac{1}{512}y_n + \frac{513}{512}y_{n+1} + \frac{3h}{1024}\left[99f_{n+1} + 80f_{n+\frac{3}{2}} - 9f_{n+2}\right] + \frac{9h^2}{2048}[7g_{n+1} + g_{n+2}]$$
$$y_{n+2} = y_{n+1} + \frac{h}{30}\left[7f_{n+1} + 16f_{n+\frac{3}{2}} + 7f_{n+2}\right] + \frac{h^2}{60}[g_{n+1} - g_{n+2}] \qquad (10)$$

CONVERGENCE AND STABILITY ANALYSIS OF THE NEW METHODS

The analysis of the newly constructed methods is carried by analyzing the order, error constant consistency, convergence and plotting the regions of absolute stability.

The local truncation error associated with (9) to be linear difference operator

$$[y(x); h] = \sum_{j=0}^{k} \alpha_j y_{n+j} - h\beta_k f_{n+k} - h^2\gamma_k g_{n+k} \ . \qquad (11)$$

Assuming that $y(x)$ is sufficiently differentiable, we can we expand the terms in (9) as a Taylor series and comparing the coefficients of h gives

$$L[y(x); h] = c_0 y(x) + c_1 h y'^{(x)} + c_2 h^2 y''^{(x)} + \cdots + c_p h^p y^p(x) + \cdots \qquad (12)$$

Where the constants $C_p, p = 0, 1,2 \ldots.., j = 1,2, \ldots k$ are given as follows:

$$C_0 = \sum_{j=0}^{k} \alpha_j$$
$$C_1 = \sum_{j=1}^{k} j\alpha_j - \sum_{j=0}^{k} \beta_j$$

. . .

. . .

$$C_q = [\frac{1}{q!}\sum_{j=0}^{k} j^q \alpha_j - \frac{1}{(q-1)!}\sum_{j=1}^{k} j^{q-1}\beta_j - \frac{1}{(q-2)!}\sum_{j=1}^{k} j^{q-2}\lambda_j \qquad (13)$$

Where

$C_0 = C_1 = C_2 \ldots C_p = C_{p+1} = 0$ and $C_{p+2} \neq 0$. Therefore, C_{p+2} is the error constant and $C_{p+2}h^{p+2}y^{(p+2)}(x_n)$ is the principal local truncation error at appoint x_n. It was established from the evaluation of both hybrid block methods have order and error constants (seeLambert,1973).

Using the concept above, the hybrid block methods are obtained with the help of MAPLE 18 SOFTWARE have the following uniform order and error constants.

The Order and Error Constants of Two-Step New Method

$$\alpha_0 = \begin{pmatrix} 1 \\ \frac{1}{512} \\ 0 \end{pmatrix}, \alpha_1 = \begin{pmatrix} -1 \\ -\frac{513}{512} \\ -1 \end{pmatrix}, \alpha_{\frac{2}{3}} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \alpha_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \beta_0 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \beta_1 = \begin{pmatrix} \frac{733}{330} \\ \frac{297}{1024} \\ \frac{7}{30} \end{pmatrix}, \beta_{\frac{2}{3}} = \begin{pmatrix} -\frac{808}{165} \\ \frac{15}{64} \\ \frac{8}{15} \end{pmatrix}, \beta_2 = \begin{pmatrix} \frac{553}{330} \\ -\frac{27}{1024} \\ \frac{7}{30} \end{pmatrix}, \gamma_1 = \begin{pmatrix} \frac{1019}{660} \\ \frac{63}{2048} \\ \frac{1}{60} \end{pmatrix}, \gamma_2 = \begin{pmatrix} -\frac{73}{220} \\ \frac{9}{2048} \\ -\frac{1}{60} \end{pmatrix}$$

Substituting the values of α, β and γ into

$$C_q = \frac{1}{q!}\left(\alpha_1 + \left(\frac{3}{2}\right)^q \alpha_{\frac{2}{3}} + 2^q\alpha_2\right) - \frac{1}{(q-1)!}\left(\beta_1 + \left(\frac{3}{2}\right)^{q-1} \beta_{\frac{2}{3}} + 2^{q-1}\beta_2\right) - \frac{1}{(q-2)!}\left(\gamma_1 + 2^{q-2}\gamma_2\right)$$

$$C_0 = \begin{pmatrix} 1 \\ \frac{1}{512} \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ -\frac{513}{512} \\ -1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$C_1 = \left(\begin{pmatrix} -1 \\ -\frac{513}{512} \\ -1 \end{pmatrix} + \left(\frac{3}{2}\right)\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + 2\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right) - \left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{733}{330} \\ \frac{297}{1024} \\ \frac{7}{30} \end{pmatrix} + \begin{pmatrix} -\frac{808}{165} \\ \frac{15}{64} \\ \frac{8}{15} \end{pmatrix} + \begin{pmatrix} \frac{553}{330} \\ -\frac{27}{1024} \\ \frac{7}{30} \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$C_2 = \frac{1}{2!}\left(\begin{pmatrix} -1 \\ -\frac{513}{512} \\ -1 \end{pmatrix} + \left(\frac{2}{3}\right)^2\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + 2^2\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right) - \left(\begin{pmatrix} \frac{733}{330} \\ \frac{297}{1024} \\ \frac{7}{30} \end{pmatrix} + \left(\frac{3}{2}\right)\begin{pmatrix} -\frac{808}{165} \\ \frac{15}{64} \\ \frac{8}{15} \end{pmatrix} + 2\begin{pmatrix} \frac{553}{330} \\ -\frac{27}{1024} \\ \frac{7}{30} \end{pmatrix}\right)$$

$$- \left(\begin{pmatrix} \frac{2}{33} \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{1019}{660} \\ \frac{63}{2048} \\ \frac{1}{60} \end{pmatrix} + \begin{pmatrix} -\frac{73}{220} \\ \frac{9}{2048} \\ -\frac{1}{60} \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$C_3 = \frac{1}{3!}\left(\begin{pmatrix} -1 \\ -\frac{513}{512} \\ -1 \end{pmatrix} + \left(\frac{2}{3}\right)^3\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + 2^3\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right) - \frac{1}{2!}\left(\begin{pmatrix} \frac{733}{330} \\ \frac{297}{1024} \\ \frac{7}{30} \end{pmatrix} + \left(\frac{2}{3}\right)^2\begin{pmatrix} -\frac{808}{165} \\ \frac{15}{64} \\ \frac{8}{15} \end{pmatrix} + 2^2\begin{pmatrix} \frac{553}{330} \\ -\frac{27}{1024} \\ \frac{7}{30} \end{pmatrix}\right)$$

$$- \left(\begin{pmatrix} \frac{1019}{660} \\ \frac{63}{2048} \\ \frac{1}{60} \end{pmatrix} + 2\begin{pmatrix} -\frac{73}{220} \\ \frac{9}{2048} \\ -\frac{1}{60} \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$C_4 = \frac{1}{4!}\left(\begin{pmatrix} -1 \\ -\frac{513}{512} \\ -1 \end{pmatrix} + \left(\frac{2}{3}\right)^4\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + 2^4\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right) - \frac{1}{3!}\left(\begin{pmatrix} \frac{733}{330} \\ \frac{297}{1024} \\ \frac{7}{30} \end{pmatrix} + \left(\frac{2}{3}\right)^3\begin{pmatrix} -\frac{808}{165} \\ \frac{15}{64} \\ \frac{8}{15} \end{pmatrix} + 2^3\begin{pmatrix} \frac{553}{330} \\ -\frac{27}{1024} \\ \frac{7}{30} \end{pmatrix}\right)$$

$$- \frac{1}{2!}\left(\begin{pmatrix} \frac{1019}{660} \\ \frac{63}{2048} \\ \frac{1}{60} \end{pmatrix} + 2^2\begin{pmatrix} -\frac{73}{220} \\ \frac{9}{2048} \\ -\frac{1}{60} \end{pmatrix}\right) = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$C_5=\frac{1}{5!}\left(\begin{pmatrix}-1\\-\frac{513}{512}\\-1\end{pmatrix}+\left(\frac{2}{3}\right)^5\begin{pmatrix}0\\1\\0\end{pmatrix}+2^5\begin{pmatrix}0\\0\\1\end{pmatrix}\right)-\frac{1}{4!}\left(\begin{pmatrix}\frac{733}{330}\\\frac{297}{1024}\\\frac{7}{30}\end{pmatrix}+\left(\frac{2}{3}\right)^4\begin{pmatrix}-\frac{808}{165}\\\frac{15}{64}\\\frac{8}{15}\end{pmatrix}+2^4\begin{pmatrix}\frac{553}{330}\\-\frac{27}{1024}\\\frac{7}{30}\end{pmatrix}\right)$$

$$-\frac{1}{3!}\left(\begin{pmatrix}\frac{1019}{660}\\\frac{63}{2048}\\\frac{1}{60}\end{pmatrix}+2^3\begin{pmatrix}-\frac{73}{220}\\\frac{9}{2048}\\-\frac{1}{60}\end{pmatrix}\right)=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$C_6=\frac{1}{6!}\left(\begin{pmatrix}-1\\-\frac{513}{512}\\-1\end{pmatrix}+\left(\frac{2}{3}\right)^6\begin{pmatrix}0\\1\\0\end{pmatrix}+2^6\begin{pmatrix}0\\0\\1\end{pmatrix}\right)-\frac{1}{5!}\left(\begin{pmatrix}\frac{733}{330}\\\frac{297}{1024}\\\frac{7}{30}\end{pmatrix}+\left(\frac{2}{3}\right)^5\begin{pmatrix}-\frac{808}{165}\\\frac{15}{64}\\\frac{8}{15}\end{pmatrix}+2^5\begin{pmatrix}\frac{553}{330}\\-\frac{27}{1024}\\\frac{7}{30}\end{pmatrix}\right)$$

$$-\frac{1}{4!}\left(\begin{pmatrix}\frac{1019}{660}\\\frac{63}{2048}\\\frac{1}{60}\end{pmatrix}+2^4\begin{pmatrix}-\frac{73}{220}\\\frac{9}{2048}\\-\frac{1}{60}\end{pmatrix}\right)=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$C_7=\frac{1}{7!}\left(\begin{pmatrix}-1\\-\frac{513}{512}\\-1\end{pmatrix}+\left(\frac{2}{3}\right)^7\begin{pmatrix}0\\1\\0\end{pmatrix}+2^7\begin{pmatrix}0\\0\\1\end{pmatrix}\right)-\frac{1}{6!}\left(\begin{pmatrix}\frac{733}{330}\\\frac{297}{1024}\\\frac{7}{30}\end{pmatrix}+\left(\frac{2}{3}\right)^6\begin{pmatrix}-\frac{808}{165}\\\frac{15}{64}\\\frac{8}{15}\end{pmatrix}+2^6\begin{pmatrix}\frac{553}{330}\\-\frac{27}{1024}\\\frac{7}{30}\end{pmatrix}\right)$$

$$-\frac{1}{5!}\left(\begin{pmatrix}\frac{1019}{660}\\\frac{63}{2048}\\\frac{1}{60}\end{pmatrix}+2^5\begin{pmatrix}-\frac{73}{220}\\\frac{9}{2048}\\-\frac{1}{60}\end{pmatrix}\right)=\begin{pmatrix}\frac{5659}{6652800}\\\frac{-9}{2293760}\\\frac{1}{604800}\end{pmatrix}$$

$C_0=C_1=C_2=C_3=C_4=C_5=C_6=0$ and $C_7\neq 0$

Thus, The New Method for $k=2$ has order $p=5$ and error constant $C_{p+2}=\begin{pmatrix}\frac{5659}{6652800}\\\frac{-9}{2293760}\\\frac{1}{604800}\end{pmatrix}$

ZERO STABILITY OF THE NEW METHOD

The two-step new method is expressed in the form of (10) gives

$$\begin{bmatrix}-1&0&0\\\frac{-513}{512}&1&0\\-1&0&1\end{bmatrix}\begin{bmatrix}y_{n+1}\\y_{n+\frac{3}{2}}\\y_{n+2}\end{bmatrix}=\begin{bmatrix}0&0&-1\\0&0&\frac{-1}{512}\\0&0&0\end{bmatrix}\begin{bmatrix}y_{n-2}\\y_{n-1}\\y_n\end{bmatrix}+h\begin{bmatrix}\frac{733}{330}&\frac{-808}{165}&\frac{553}{330}\\\frac{297}{1024}&\frac{15}{65}&\frac{-27}{1024}\\\frac{7}{30}&\frac{8}{15}&\frac{7}{30}\end{bmatrix}\begin{bmatrix}f_{n+1}\\f_{n+\frac{3}{2}}\\f_{n+1}\end{bmatrix}+h^2\begin{bmatrix}\frac{1019}{660}&0&\frac{-73}{220}\\\frac{63}{1024}&0&\frac{9}{2048}\\\frac{1}{60}&0&\frac{-1}{60}\end{bmatrix}\begin{bmatrix}g_{n+1}\\g_{n+\frac{3}{2}}\\g_{n+1}\end{bmatrix}$$

Where $A=\begin{bmatrix}-1&0&0\\\frac{-513}{512}&1&0\\-1&0&1\end{bmatrix}$, $B=\begin{bmatrix}0&0&-1\\0&0&\frac{-1}{512}\\0&0&0\end{bmatrix}$, $C=\begin{bmatrix}\frac{733}{330}&\frac{-808}{165}&\frac{553}{330}\\\frac{297}{1024}&\frac{15}{65}&\frac{-27}{1024}\\\frac{7}{30}&\frac{8}{15}&\frac{7}{30}\end{bmatrix}$,

$$D = \begin{bmatrix} \frac{1019}{660} & 0 & \frac{-73}{220} \\ \frac{63}{1024} & 0 & \frac{9}{2048} \\ \frac{1}{60} & 0 & \frac{-1}{60} \end{bmatrix},$$

Substituting A, B, C and D in to

$$det[r(A - Cz - Dz^2 - B)] = 0 \qquad (14)$$

using Maple yields the stability polynomial of the method as

$$\rho(r) = \det(rA - B)$$

$$\rho(r) = det\left[r\begin{bmatrix} -1 & 0 & 0 \\ -\frac{513}{512} & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & -\frac{1}{512} \\ 0 & 0 & 0 \end{bmatrix} \right]$$

$$= det\begin{bmatrix} -r & 0 & 1 \\ -\frac{513}{512} & r & \frac{1}{512} \\ -z & 0 & r \end{bmatrix}$$

$= r^2(r-1) = 0$

$r_1 = 1, r_2 = r_3 = 0 < 1$

Hence, the block method (10) is zero stable and is of order $P = 5$ and hence by Henrici (1962) it is convergent.

ABSOLUTE STABILITY REGION OF THE NEW METHOD

The coefficient of the block method (10) expressed in the form

$$\begin{bmatrix} -1 & 0 & 0 \\ \frac{-513}{512} & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} y_{n+1} \\ y_{n+\frac{3}{2}} \\ y_{n+2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & \frac{-1}{512} \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} y_{n-2} \\ y_{n-1} \\ y_n \end{bmatrix} + h\begin{bmatrix} \frac{733}{330} & \frac{-808}{165} & \frac{553}{330} \\ \frac{297}{1024} & \frac{15}{65} & \frac{-27}{1024} \\ \frac{7}{30} & \frac{8}{15} & \frac{7}{30} \end{bmatrix}\begin{bmatrix} f_{n+1} \\ f_{n+\frac{3}{2}} \\ f_{n+1} \end{bmatrix} + h^2\begin{bmatrix} \frac{1019}{660} & 0 & \frac{-73}{220} \\ \frac{63}{1024} & 0 & \frac{9}{2048} \\ \frac{1}{60} & 0 & \frac{-1}{60} \end{bmatrix}\begin{bmatrix} g_{n+1} \\ g_{n+\frac{3}{2}} \\ g_{n+1} \end{bmatrix}$$

Where $A = \begin{bmatrix} -1 & 0 & 0 \\ \frac{-513}{512} & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & \frac{-1}{512} \\ 0 & 0 & 0 \end{bmatrix}, C = \begin{bmatrix} \frac{733}{330} & \frac{-808}{165} & \frac{553}{330} \\ \frac{297}{1024} & \frac{15}{65} & \frac{-27}{1024} \\ \frac{7}{30} & \frac{8}{15} & \frac{7}{30} \end{bmatrix},$

$$D = \begin{bmatrix} \frac{1019}{660} & 0 & \frac{-73}{220} \\ \frac{63}{1024} & 0 & \frac{9}{2048} \\ \frac{1}{60} & 0 & \frac{-1}{60} \end{bmatrix},$$

Substituting the values of A, B, C, D and $z = \lambda h$ into (14) we obtain

The stability function $R(z)$ is given by

$$R(z) = \frac{23r}{44} + \frac{37r^2z}{165} + \frac{17r^2z^2}{528} + \frac{65}{44}r^3 - \frac{332r^3z}{165} + \frac{99r^3z^2}{80} - \frac{431r^3z^3}{990} + \frac{r^3z^4}{12}$$

And substituting $D(z) and R(z)$ into a MATLAB code for plotting stability regions produces the absolute stability regions of the two-step block method.

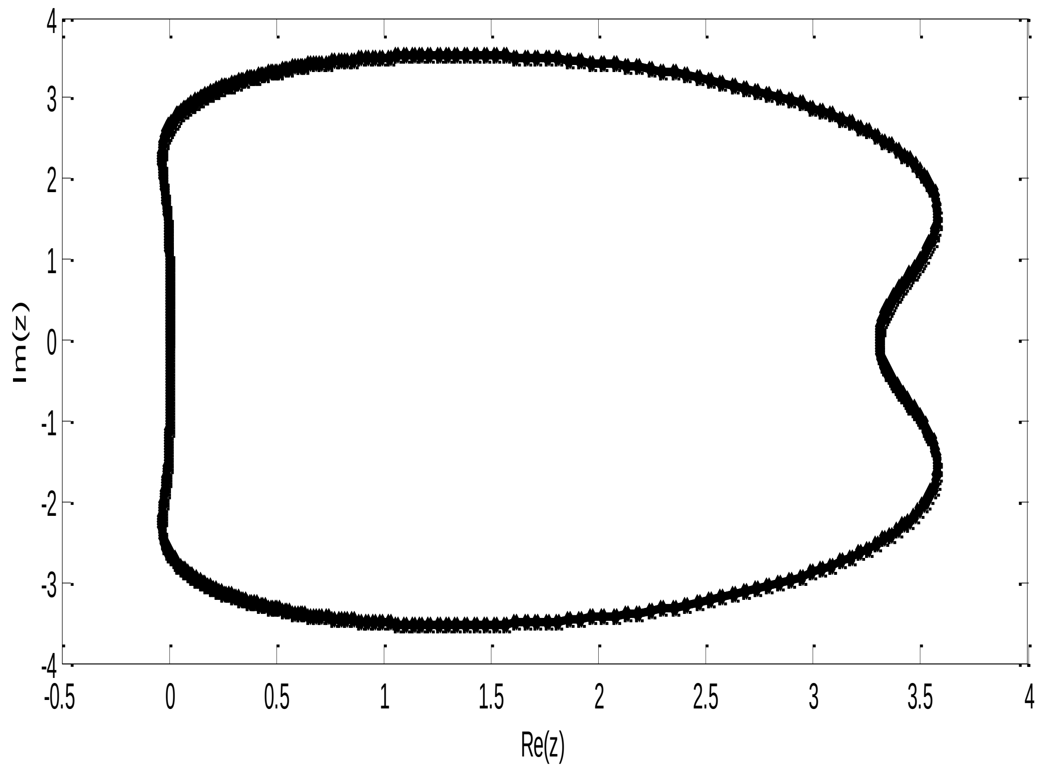

Figure 1. Region of absolute stability of New Methods for k=2

Therefore, the block method is $A - stable$.

NUMERICAL EXAMPLE

We used the newly constructed method called two-step hybrid block backward differentiation formulae to solve the following nonlinear initial value problems.

Problem 1

$$y_1' = -2.000001y_1 + 0.000001y_2^2$$

$$y_2' = y_1 - y_2 - y_2^2 \qquad with\ h = 0.01$$

Exact $y_{1(x)} = e^{-2x} \quad y_1(0) = 1$

$y_2(x) = e^{-x} \quad y_2(0) = 1$

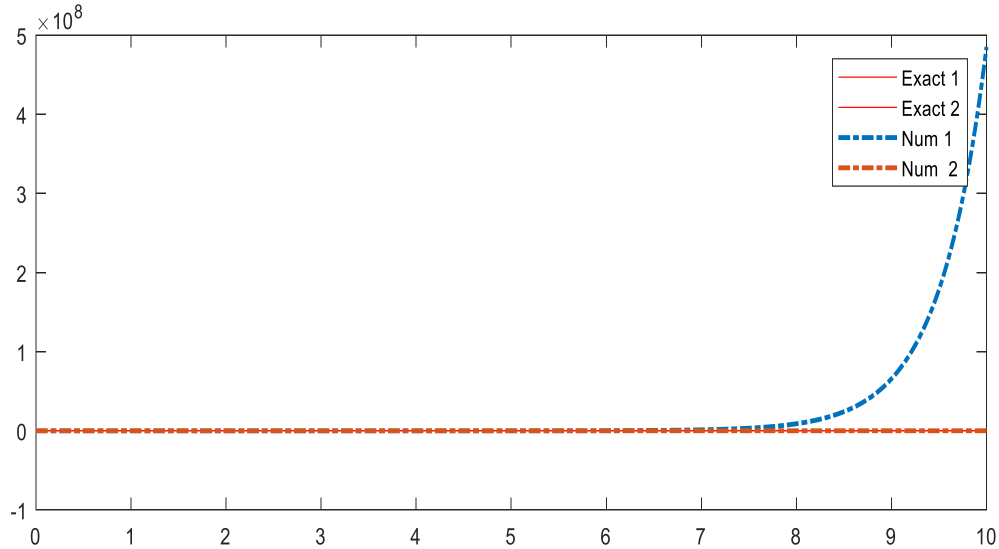

Figure 2: Solution Curve of Problem 1 Solved with Two-step Method

Problem 2

Predator- pre

$$y_1' = 1.2y_1 - 0.6y_1y_2 \quad y_1(0) = 2$$
$$y_2' = -0.8y_2 + 0.3y_1y_2 \quad y_2(0) = 1$$
$$with\ h = 0.01$$

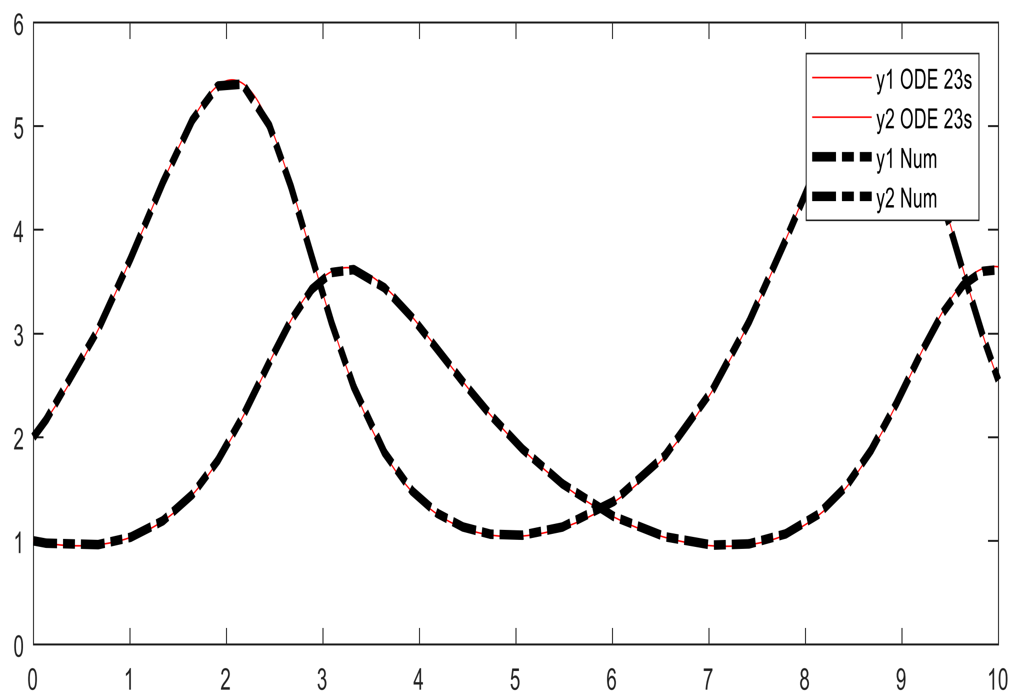

Figure 3: Solution Curve of Problem 2 Solved with Two-step Method

CONCLUSIONS

In the course of this research, the multistep collocation approach was used in the derivation of two-step hybrid block backward differentiation formulae for the solution of stiff ordinary differential equations. The method was successfully implemented nonlinear stiff ordinary differential equations and methods was proven to be consistent, zero stable and convergent. The region of absolute stability of the method shows that is A-stable. The curve solution results obtained in this research showed that the derived schemes solve satisfactorily the problems in nonlinear stiff differential equations and performed better in the approximation of the Predator- prey model when compared with the well-known ODE solver, ODE 23s.

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